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Innovation and horizontal mergers in a vertically related industry*

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Abstract

This paper analyses the effects of horizontal mergers on innovation and consumer welfare in a vertically related industry context, in which downstream firms compete for customers with a differentiated final good and can undertake R&D activities to reduce their unit costs. Upstream and downstream horizontal mergers can take place.

The results suggest that competition authorities aiming to promote innovation and consumer welfare should treat upstream and downstream mergers differently, since horizontal mergers between upstream firms are detrimental to innovation and consumer welfare. By contrast, policy makers should evaluate the market characteristics under downstream integration. We show that downstream horizontal mergers can be both innovation and consumer welfare enhancing in the short run, when the markets are sufficiently small.

Keywords: Horizontal Mergers. Innovation. Vertical Relations.

JEL Classification Numbers: L22, L41, O32

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1 Introduction

Innovation is a key issue for competitiveness, both for countries and for companies (Veugelers, 2012). For example, the EU includes research and innovation as fundamental aspects of 'smart, sustainable and inclusive growth' in the Horizon 2020 strategy (European Commission, 2014); the American Strategy for Innovation reckons that 'America's future economic growth and international competitiveness depend on our capacity to innovate' (White House, 2014). In this context, firms often argue that mergers and acquisitions (M&As) constitute leverage for innovation. However, it is well known that M&As may have many undesirable repercussions. Particularly, horizontal mergers reduce competition and constitute a major concern for competition authorities (Röller et al., 2000). Therefore, it is especially relevant to understand and assess the consequences of M&As for the innovative potential of firms.

The European Commission in its Guidelines on the Assessment of Horizontal Mergers (2004) includes the evaluation of R&D and innovation effects. Similarly, the US Department of Justice and the Federal Trade Commission consider innovation effects in their Horizontal Merger Guidelines (2010).[1] In practice, however, innovation effects in horizontal mergers are difficult to assess due to their intrinsic uncertainty (Shapiro, 2010). Consequently, the European Commission only takes them into account when claimed by the parties involved in a merger. Until now, innovation-related potential advantages have not been decisive in the prohibition or allowance of horizontal mergers (Veugelers, 2012). In the US, R&D and innovation effects also play a modest role in merger evaluation decisions. However, the US Antitrust Modernization Commission has recommended giving greater weight to R&D efficiencies (Katz and Shelanski, 2007).

The difficulty in evaluating the effects of mergers on innovation might also come from the lack of a direct analysis of the link between innovation and mergers in the literature. Only Cassiman et al. (2005) provide some empirical evidence, concluding that there is a positive effect in the case of technologically complementary M&As. Thus, there is no theoretical analysis studying the direct impact of horizontal mergers on R&D. Instead, the relationship between mergers and R&D has been evaluated indirectly as, on the one hand, the effect of mergers on competition and, on the other hand, the impact of competition on innovation, are rather well understood (Veugelers, 2012). On this ground, this paper provides a framework to study the consequences of upstream and downstream horizontal mergers for firms' innovation decisions.

We model a vertically related industry with an upstream and a downstream duopoly.[2] Upstream firms supply a homogeneous input to downstream firms, which produce a differentiated final good. In our setting, a downstream firm can reduce its unit cost by undertaking process innovation,[3] stemming from its R&D effort and the spillovers derived from the other firm's R&D. The innovation decision is reflected by a binary variable, which represents whether a downstream firm engages in R&D activities or not. The R&D efforts are costly and reduce the downstream profits. We consider three different market structures: *i*) the Base Case without mergers, *ii*) Upstream Integration, and *iii*) Downstream Integration.

Our main findings are as follows. Although downstream firms' R&D efforts reduce the unit costs, they also produce an increase in wholesale prices, which discourages innovation. The equilibrium in innovation strategies

[1] The most recent updated version of the Horizontal Merger Guidelines (2010), Section 6.4., includes an analysis of innovation and product variety as entirely new aspects to be evaluated.

[2] According to Banerjee and Lin (2003), much corporate R&D is conducted in a supplier-customer context.

[3] Von Hippel (1988) finds that more than two-thirds of first-to-market innovations are dominated by end-users. Retailers engage in direct interactions with end customers, unlike most manufacturers, which let them capture their changing needs and offer innovative products and services (Sorescu et al., 2011).

depends on the market structure. Thus, horizontal mergers can affect firms' innovation decisions substantially. We conclude that Upstream Integration has a negative effect on innovation, while Downstream Integration has a positive effect. Regarding consumer welfare, we show that, for a given innovation strategy across scenarios, consumer welfare is maximized in the absence of mergers. In addition, we confirm that upstream mergers are detrimental to consumer welfare. In contrast, and most interestingly, downstream mergers can be consumer welfare enhancing when the markets are sufficiently small.

The paper is organized as follows. The literature review is presented in Section 2. In Section 3, we present the three scenarios considered: the Base Case (Subsection 3.1), Upstream Integration (Subsection 3.2), and Downstream Integration (Subsection 3.3). Section 4 studies the implications of horizontal mergers for consumer welfare. Finally, Section 5 presents some policy implications and closes the paper. The positivity, second-order and stability conditions and all the proofs are provided in the Appendix.

2 Literature Review

In the literature, the direct effects of mergers on innovation have not been properly examined.[4] Only Cassiman et al. (2005) provide some empirical evidence, concluding that there is a positive effect of M&As on innovation in the case of technologically-complementary firms and non-rival firms. We try to fill this gap by providing a theoretical framework to study the consequences of upstream and downstream horizontal mergers for firms' innovation decisions. We connect two strands of the literature. On the one hand, only Banerjee and Lin (2003) connect vertical relations and innovation and, on the other hand, a branch of the literature analyses horizontal mergers in a vertically related industry, distinguishing between downstream and upstream mergers.

More precisely, Banerjee and Lin (2003) study innovation in the presence of vertical relations. The authors present a theoretical model in which a monopoly upstream firm supplies an intermediate good to several downstream firms. The Downstream firms can reduce their marginal unit cost by undertaking R&D activities. As a result, the upstream monopoly increases the input price, affecting the downstream firms' incentive to innovate. Fixed-price contracts between upstream and downstream firms can be a means of controlling the downstream production costs and stimulating innovation at this level.

Regarding the branch of the literature analysing horizontal mergers in a vertically-related industry and focusing on downstream mergers, von Ungern-Sternberg (1996) finds that they lead to a reduction in the final price when retailers act as price takers. In the same vein, Dobson and Waterson (1997) obtain a similar result when retail services are regarded as very close substitutes. From a different perspective, Lommerud et al. (2005) and Faulí-Oller and Sandonis (2010) analyse the merger consequences for input prices. Lommerud et al. (2005) find that potential merger partners at the downstream level should take into account the presence of market power at the upstream level. Faulí-Oller and Sandonis (2010) show that downstream mergers lead to lower wholesale prices, which translate into lower final prices when there are two sufficiently differentiated suppliers in terms of efficiency. Finally, Symeonidis (2010) finds that downstream mergers may raise the consumer surplus and overall welfare when there is quantity competition, upstream agents are independent, and bargaining is over a uniform input price.[5]

[4] Cassiman et al. (2005) present an extensive literature review on financial economics, industrial organization, and strategic and technology management, which gives many indirect insights into the relationship between M&As and R&D.

[5] The opposite result is obtained when there is price competition, the upstream agents are not independent, the bargaining is

Considering upstream mergers, Milliou and Petrakis (2007) show that upstream firms prefer to merge (not to merge) under wholesale-price contracts (two-part tariff contracts). They conclude that upstream horizontal mergers should not be allowed, although they can generate some efficiency gains.

A couple of papers study mergers both at the upstream and at the downstream level. First, Ziss (1995) shows that upstream mergers are typically anti-competitive under very general demand and cost conditions, whereas downstream mergers are pro-competitive both in the presence and in the absence of intra-brand competition, and for both price and output competition at the downstream level. Second, Horn and Wolinsky (1988) demonstrate that merger incentives are present when the downstream firms compete in the final goods' market.

3 The Model

Consider an industry with two upstream and two downstream firms denoted by U_i and D_i, with $i = 1, 2$, respectively. Each upstream firm produces a homogenous input which is transformed by downstream firms into a final good in a one-to-one proportion. An exclusive upstream-downstream relationship is assumed, such that D_i purchases its input only from U_i.

The downstream demand functions are derived from the maximization of the welfare function of a representative consumer (as in Bowley, 1924)

$$U = u(M) + aq_i + aq_j - \frac{1}{2}\left[\left(q_i^2 + q_j^2\right) + 2dq_iq_j\right], \ \ i, j = 1, 2, \ \ i \neq j,$$

subject to the budget constraint

$$Y = M + p_iq_i + p_jq_j,$$

where q_i is the quantity and p_i is the price of good i, M is the quantity of other goods consumed, Y denotes income, parameter $d \in (0, 1)$ measures the degree of product differentiation,[6] and $a > 0$. The resulting downstream demand function is

$$q_i = \frac{a}{1+d} - \frac{1}{1-d^2}p_i + \frac{d}{1-d^2}p_j. \tag{1}$$

When the products are independent (i.e., $d = 0$), D_i is a monopolist in its market. As the products become closer substitutes (i.e., as d increases), the competition intensity in the downstream market increases.

The profits of D_i are given by

$$\pi_{Di}(S_i, S_j) = (p_i - p_{wi} - c_i)\, q_i - \epsilon x_i, \tag{2}$$

where p_{wi} is the wholesale price that D_i pays per unit of input to U_i, and $c_i = c - \gamma x_i - \sigma\gamma x_j$ is D_i's cost function. The unit production cost c is reduced by γx_i, where $x_i \in \{0, 1\}$ is a binary variable that denotes the firm's own R&D effort with $\gamma \in [0, 1]$ and $\sigma\gamma x_j$ stands for the spillovers stemming from the competitor's

over a two-part tariff, or the bargaining covers both the input price and the level of output.

[6] The unilateral effects, the role of price/cost margins, and market definition are three related issues that become relevant when a merge proposal is investigated in markets with differentiated products (Shapiro, 2010). The level of product differentiation is relevant when a merger proposal is evaluated. More precisely, in the 2010 Guidelines, Sections 6.1 and 6.2 address the pricing and bidding competition among suppliers of differentiated products.

innovation effort x_j with $\sigma \in [0,1]$ being the spillover intensity parameter.[7] Finally, ϵ is the unit cost of innovation with $\epsilon > 0$. We assume that $c \geq \gamma(1+\sigma)$, such that $c_i \geq 0$.

The profits of U_i are given by

$$\pi_{Ui} = (p_{wi} - c_U)\, q_{U_i}, \tag{3}$$

where c_U is the upstream unit cost and $q_{U_i} = q_i$ because inputs are transformed into outputs in a one-to-one proportion.

The timing of the game is as follows. In Stage 1, the downstream firms decide whether to innovate or not by choosing strategy $x_i \in \{1, 0\}$, where $x_i = 1$ indicates that D_i innovates, and $x_i = 0$ that D_i does not innovate. In Stage 2, the upstream firms maximize equation (3) by choosing their wholesale prices p_{wi}. Finally, in Stage 3, the downstream firms choose simultaneously and independently p_i to maximize equation (2). The game is solved by backward induction.

3.1 The Base Case

In the absence of mergers, the Stage-3 profit maximization yields the following reaction function

$$p_i = \frac{1}{2}\left[(1-d)\,a + c_i + dp_j + p_{wi}\right]. \tag{4}$$

D_i's price depends positively on the wholesale price p_{wi} and on its competitor's price p_j. The equilibrium price and output are

$$p_i = \frac{(2+d)(1-d)\,a + 2(c_i + p_{wi}) + d(c_j + p_{wj})}{4-d^2}, \tag{5}$$

$$q_i = \frac{(2+d)(1-d)\,a - \left(2-d^2\right)(c_i + p_{wi}) + d(c_j + p_{wj})}{(4-d^2)(1-d^2)}. \tag{6}$$

As the products become closer substitutes (i.e., as $d \to 1$), firm j's unit cost and wholesale price affect firm i's price and output positively.

In Stage 2, the upstream firms maximize equation (3) by choosing their wholesale prices p_{wi}. The upstream firms' reaction functions are

$$p_{wi} = \frac{(2+d)(1-d)\,a + \left(2-d^2\right)(c_U - c_i) + d(p_{wj} + c_j)}{2(2-d^2)}. \tag{7}$$

We observe that the wholesale prices are strategic complements as long as $d > 0$. The equilibrium wholesale price is

$$p^0_{wi}(x_i, x_j) = \frac{1}{4}\left[2(a + c_U - c_i) - \frac{d(2a - 2c_U - c_i - c_j)}{4 - d - 2d^2} - \frac{d(c_i - c_j)}{4 + d - 2d^2}\right], \tag{8}$$

where superindex 0 makes reference to the Base Case. Regarding the effect of R&D effort on the wholesale prices in equation (8) we obtain the following result.

Lemma 1 *Firm i's R&D effort produces an increase in its own wholesale price p^0_{wi}, as well as in its competitor's wholesale price p^0_{wj}. The positive effects of D_i's R&D effort on both wholesale prices decreases with product differentiation.*

[7] In some related literature, the intensity of spillovers is associated with the protection of the intellectual property. High (low) values of spillovers mean that there is weak (strict) protection of intellectual property.

An increase in D_i's R&D effort reduces its own unit cost c_i, which raises D_i's profits. As a consequence, U_i increases its wholesale price to extract part of these additional profits. Since the wholesale prices are strategic complements, U_j also increases its wholesale price.[8] As the products become closer substitutes, the downstream prices fall and the scope for further price reductions is more limited. Thus, the effect of the downstream R&D effort on the wholesale prices decreases.

Looking at equation (4), we can now identify two differentiated effects of R&D efforts on consumer prices. On the one hand, there is a direct negative effect: R&D effort reduces the unit costs, which is transferred to a price reduction. On the other hand, there is an indirect positive effect: R&D effort increases the wholesale prices (as shown in Lemma 1), which in turn yield an increase in the consumer prices.

Substituting equation (8) into equations (5) and (6), we obtain the Stage-2 Nash equilibrium downstream price and output

$$p_i^0(x_i,x_j) = \frac{2(1-d)\left(3-d^2\right)a+\left(2-d^2\right)c_U}{(2-d)\left(4-d-2d^2\right)} + \frac{\left(2-d^2\right)\left[\left(8-3d^2\right)c_i+2d\left(3-d^2\right)c_j\right]}{\left(4-d^2\right)\left(4+d-2d^2\right)\left(4-d-2d^2\right)}, \tag{9}$$

$$q_i^0(x_i,x_j) = \frac{\left(2-d^2\right)(a-c_U)}{(2-d)(1+d)\left(4-d-2d^2\right)} - \frac{\left(2-d^2\right)\left[\left(8-9d^2+2d^4\right)c_i-d\left(2-d^2\right)c_j\right]}{\left(4-d^2\right)\left(1-d^2\right)\left(4+d-2d^2\right)\left(4-d-2d^2\right)}, \tag{10}$$

and plugging these values into equation (2), we obtain the SPNE downstream and upstream profits

$$\pi_{Di}^0(x_i,x_j) = \left(1-d^2\right)q_i^0(x_i,x_j)^2-\epsilon x_i, \tag{11}$$

$$\pi_{Ui}^0(x_i,x_j) = q_i^0(x_i,x_j)^2\,\frac{\left(4-d^2\right)\left(1-d^2\right)}{\left(2-d^2\right)}. \tag{12}$$

From the above expressions, we can observe that the upstream and downstream profits decrease as the products become closer substitutes. For the ensuing analysis, let us denote $W \equiv (a-c-c_U)$ and make the following assumption, which ensures positivity of the prices and outputs (see Appendix A for further details). For the sake of simplicity and to simplify the notation, we will refer to W as the market size, although it also comprises the downstream and upstream unit costs.

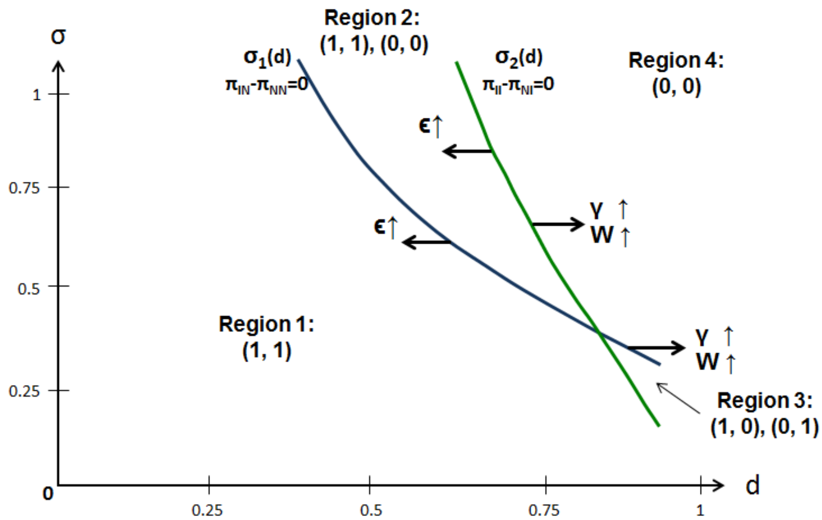
Assumption 1 Let $W > \underline{W} \equiv \frac{\gamma(d-\sigma)}{(1-d)}$ delimit the relevant region in our analysis.

The condition in Assumption 1 comes from the positivity conditions, which are thoroughly explained in Appendix A. In Stage 1, the downstream firms choose their R&D strategy to maximize their profits. We compare the equilibrium downstream profits $\pi_i^0(x_i,x_j)$ under the three innovation strategies considered, i.e., when both downstream firms innovate, when neither of them innovates, and when only one of them innovates.

Considering the possible innovation strategies that can be chosen by the downstream firms, we may have four equilibrium regions, which are delimited by functions $\sigma_1(d)$ and $\sigma_2(d)$ in Fig.1, which are implicitly defined by $\pi_i^0(1,0)-\pi_i^0(0,0)=0$, and $\pi_i^0(1,1)-\pi_i^0(0,1)=0$, respectively. The function $\sigma_1(d)$ is obtained from D_i's unilateral incentive to innovate, and $\sigma_2(d)$ is obtained from D_i's incentive to innovate when D_j also innovates. In Region 1, both firms engage in innovation strategies. This equilibrium occurs for sufficiently low values of product differentiation (d) and spillover intensity (σ). Multiple equilibria arise in Region 2 and Region 3. On the one hand, in Region 2, the downstream firms follow symmetric innovation strategies and either both of

[8] The fact that U_j increases its wholesale price due to D_i's R&D effort is known as a rising rival cost effect, as noted by Banerjee and Lin (2003).

them innovate or neither of them innovates. On the other hand, in Region 3, we have two asymmetric equilibria in which only one firm innovates. Finally, in Region 4, no firm innovates.



Note: Parameter values: a=13, c_U=3, c=2, γ=1, and ϵ=0.9.

Fig. 1. Innovation equilibria in the base case.

Lemma 2 below specifies the conditions under which the different innovation equilibria exist.

Lemma 2 *In the Base Case, the equilibria in innovation strategies are characterized as follows:*
i) for $\sigma(d) < \min\{\sigma_1(d), \sigma_2(d)\}$*, both firms engage in innovation (Region 1),*
ii) for $\sigma_1(d) < \sigma(d) < \sigma_2(d)$*, there are symmetric multiple equilibria of the type* $\{(1,1), (0,0)\}$ *(Region 2),*
iii) for $\sigma_2(d) < \sigma(d) < \sigma_1(d)$*, there are asymmetric multiple equilibria of the type* $\{(1,0), (0,1)\}$ *(Region 3),*
iv) for $\sigma(d) > \max\{\sigma_1(d), \sigma_2(d)\}$*, neither firm innovates (Region 4).*
We have $d\sigma_i/dW > 0$*,* $d\sigma_i/d\gamma > 0$*, and* $d\sigma_i/d\epsilon < 0$*, with* $i = 1, 2$ *for* $W > W^*$*.*

Lemma 2 suggests that firms optimally decide to innovate (not to innovate) when the spillovers are low (high) and the products are sufficiently independent (close substitutes).[9] On the one hand, the effect of d on innovation is favourable as pointed out in Lemma 1. On the other hand, the effect of σ is negative since firm i can benefit from firm j's R&D effort. For high values of d and σ, we observe that the net effect is negative, giving rise to an equilibrium without innovation.

Examining the effect of the parameters on $\sigma_1(d)$ and $\sigma_2(d)$, a comparative static exercise can be conducted, in which the effects obtained are as expected. When the cost of innovation (ϵ) increases, the region where both downstream firms innovate (do not innovate) shrinks (expands). Differently, the market size (W) and the marginal benefit of R&D (γ) have a positive effect on innovation. In the next section, we consider an upstream horizontal merger while the downstream duopoly remains and we study its effects on the equilibrium innovation strategies.

[9] There are two regions with multiple equilibria. Thus, there is a parsimonious transition between the two regions with a unique equilibrium.

3.2 Upstream Integration

Having explained the Base Case, our attention now shifts to the Upstream Integration scenario, in which upstream firms form a monopoly U, which produces a homogenous input to downstream firms. Consequently, the upstream demand is given by

$$q_U = \sum_{i=1,2} q_i \tag{13}$$

and, therefore, the upstream profit function becomes

$$\pi_U = \sum_{i=1,2} (p_{wi} - c_U)\, q_i. \tag{14}$$

Stage 3 is as in the Base Case. In Stage 2, the integrated upstream firm chooses its profit-maximizing wholesale prices p_{wi}, which are given by

$$p_{wi}^{T}(x_i, x_j) = \frac{1}{2}(a + c_U - c_i), \tag{15}$$

where superindex T makes reference to a merger at the *top* (upstream merger). Interestingly, unlike the Base Case, the wholesale price depends neither on product differentiation nor on the competitor's unit cost (c_j). The reason is that, now, the upstream monopoly has the market power to determine both wholesale prices and, thus, the level of product differentiation is irrelevant. Substituting equation (15) into equations (5) and (6), we obtain the Stage-2 Nash equilibrium downstream prices and outputs:

$$p_i^{T}(x_i, x_j) = \frac{(2+d)\left[a(3-2d) + c_U\right] + 2c_i + dc_j}{2(2-d)(2+d)}, \tag{16}$$

$$q_i^{T}(x_i, x_j) = \frac{(2+d)(1-d)(a-c_U) - \left(2-d^2\right)c_i + dc_j}{2(2-d)(2+d)(1-d)(1+d)}. \tag{17}$$

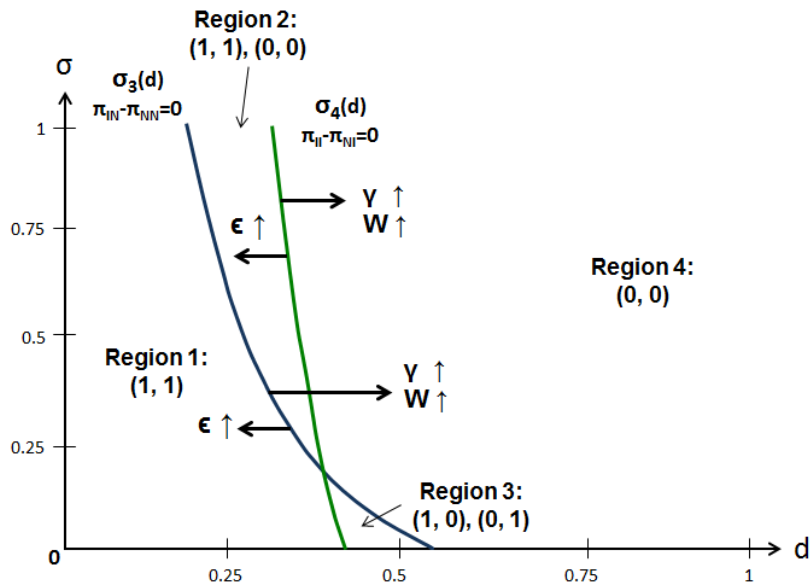
Notice that, as the products become increasingly independent (i.e., as $d \to 0$), the downstream firms become unrelated; therefore, the price and the output converge to the ones under the Base Case, in which there are exclusive relationships between the upstream and the downstream firms. The Stage-2 downstream and upstream profits, respectively, can be expressed as a function of the output:

$$\pi_{Di}^{T}(x_i, x_j) = \left(1-d^2\right) q_i^{T}(x_i, x_j)^2 - x_i \epsilon, \tag{18}$$

$$\pi_U^{T}(x_i, x_j) = \sum_{i=1,2} \left[\left(p_{wi}^{T}(x_i, x_j) - c_U\right) q_i^{T}(x_i, x_j)\right]. \tag{19}$$

From equation (18) we can observe that the expression is similar to that obtained in the Base Case (i.e., profits decrease with d) and the two scenarios converge as $d \to 0$.

The Stage-1 equilibrium analysis in innovation strategies under Upstream Integration gives rise to four equilibrium regions (as before), which are shown in Fig. 2. Functions $\sigma_3(d)$ and $\sigma_4(d)$ correspond to functions $\sigma_1(d)$ and $\sigma_2(d)$ in Fig. 1. The difference with respect to the Base Case is that the functions have moved leftward.



Note: Parameter values: a=13, c_U=3, c=2, γ=1, and ϵ=0.9.

Fig. 2. Innovation equilibria under upstream integration.

We now obtain Lemma 3, which specifies the conditions for the different innovation equilibria to arise.

Lemma 3 *In R, for given parameters values* $\{a,\ c,\ c_U,\ \gamma,\ \epsilon\}$*, the equilibria in innovation strategies are characterized as follows:*
i) for $\sigma(d) < \min\{\sigma_3(d), \sigma_4(d)\}$*, both firms engage in innovation* (*Region* 1)*,*
ii) for $\sigma_3(d) < \sigma(d) < \sigma_4(d)$*, there are multiple equilibria of the type* $(1,1)$ *and* $(0,0)$ (*Region 2*)*,*
iii) for $\sigma_4(d) < \sigma(d) < \sigma_3(d)$*, there are asymmetric equilibria of the type* $(1,0)$ *and* $(0,1)$ (*Region 3*)*,*
iv) for $\sigma(d) > \max\{\sigma_3(d), \sigma_4(d)\}$*, neither firm innovates* (*Region 4*)*.*
We have $d\sigma_i/dW > 0$*,* $d\sigma_i/d\gamma > 0$*, and* $d\sigma_i/d\epsilon < 0$*, with* $i = 3, 4$ *for* $W > W^{\blacktriangle}$*.*

From the comparison between Fig. 1 and Fig. 2, we observe that the region where both downstream firms engage in innovation is reduced, while the region where no downstream firm innovates increases. The proposition that follows arises from the comparison of the results in Lemma 2 and Lemma 3.

Proposition 1 *Upstream integration has a negative effect on innovation.*

Thus, downstream firms have fewer incentives to innovate in the presence of an upstream monopoly. This is explained by the fact that the wholesale prices in scenario T are greater than those in scenario 0, i.e., $p^T_{wi}(x_i, x_j) > p^0_{wi}(x_i, x_j)$, and the negative effect of innovation on the wholesale prices is now greater.[10] The comparative-static effects of parameters W, γ, and ϵ are as in the Base Case. This result justifies the systematic concerns of competition authorities regarding horizontal mergers between upstream firms.[11] In the

[10] It can be checked that $\frac{\partial p^0_{wi}}{\partial c_i}\frac{\partial c_i}{\partial x_i} < \frac{\partial p^T_{wi}}{\partial c_i}\frac{\partial c_i}{\partial x_i}$, where $\frac{\partial p^0_{wi}}{\partial c_i}\frac{\partial c_i}{\partial x_i} = \frac{\gamma}{4}\left(2 + \frac{d}{4+d-2d^2} - \frac{d}{4-d-2d^2}\right)$ and $\frac{\partial p^T_{wi}}{\partial c_i}\frac{\partial c_i}{\partial x_i} = \frac{\gamma}{2}$.

[11] The 1982 Guidelines were written with relatively homogeneous, industrial products in mind. This reflects the longstanding antitrust concerns about the performance of concentrated markets for basic industrial commodities (Shapiro, 2010).

next scenario, we analyse the implications for the equilibrium innovation strategies of a downstream merger (while the upstream duopoly remains).

3.3 Downstream Integration

Under Downstream Integration, the merged entity becomes a multiproduct monopoly that transforms inputs into two differentiated products.[12] Thus, the downstream profit function is now defined as

$$\pi_D = \sum_{i=1,2} \left[(p_i - p_{wi} - c_i) q_i - \epsilon x_i \right], \tag{20}$$

since there is a unique decision-maker.

In Stage 3, the downstream firm D maximizes its profits by choosing prices p_1 and p_2. The Stage-3 Nash equilibrium prices and output are given by

$$p_i = \frac{1}{2}(a + c_i + p_{wi}), \tag{21}$$

$$q_i = \frac{(1-d)a - c_i + p_{wi} - d(c_j + p_{wj})}{2(1-d)(1+d)}. \tag{22}$$

When the products are independent, having a multimarket monopoly is tantamount to having two independent monopolies. Therefore, the price in equation (21) is the same as that in the Base Case (equation (5)) for $d = 0$.

In Stage 2, the upstream firms maximize their profits in equation (3) by choosing p_{wi}. The Stage-2 wholesale prices are given by

$$p_{wi}^B(x_i, x_j) = \frac{(2+d)\left[(1-d)a + c_U\right] - c_i\left(2-d^2\right) + dc_j}{(2+d)(2-d)}, \tag{23}$$

where superindex B makes reference to a merger at the *bottom* (downstream merger).

It emerges that the Stage-2 prices and output are equal to those under Upstream Integration (see equations (16) and (17)). The upstream and downstream profits are now

$$\pi_D^B(x_i, x_j) = \sum_{i,j=1,2} \left[\frac{(2+d)(a-c_U) - 2c_i - dc_j}{2(2-d)(2+d)} q_i^B - \epsilon x_i \right], \quad i,j = 1,2, \quad i \neq j, \tag{24}$$

$$\pi_{Ui}^B(x_i, x_j) = 2\left(1-d^2\right)\left[q_i^B(x_i, x_j)\right]^2. \tag{25}$$

In Stage 1, the downstream monopolist firm chooses its R&D strategy regarding its two products. We compare the equilibrium profits $\pi_D^B(x_i, x_j)$ under the three innovation strategies considered, i.e., to innovate in both products simultaneously, $(1,1)$, to innovate just in one product, either $(1,0)$ or $(0,1)$, or not to innovate in either product $(0,0)$. Considering the possible innovation strategies that can be chosen by the downstream firm, we may have different equilibria delimited by functions $\sigma_5(d)$, $\sigma_6(d)$, and $\sigma_7(d)$, which are implicitly defined by $\pi_D^B(1,1) - \pi_D^B(0,0) = 0$, $\pi_D^B(1,1) - \pi_D^B(1,0) = 0$, and $\pi_D^B(1,0) - \pi_D^B(0,0) = 0$, respectively. In Region 1, for sufficiently high values of d and σ, the downstream firm innovates in both products. In Region 2, for sufficiently low values of product differentiation and spillover intensity, there is no innovation in either

[12] An alternative assumption would be to consider that the integrated firm just produces an homogeneous output. However, this would require the adjustment of the production process in at least one of the plants, which might involve additional costs (e.g., because of specific investments in each of the plants).

of the two products. Finally, in Region 3, when the products are close substitutes and there is a low level of spillover intensity, the downstream monopoly innovates in just one product.

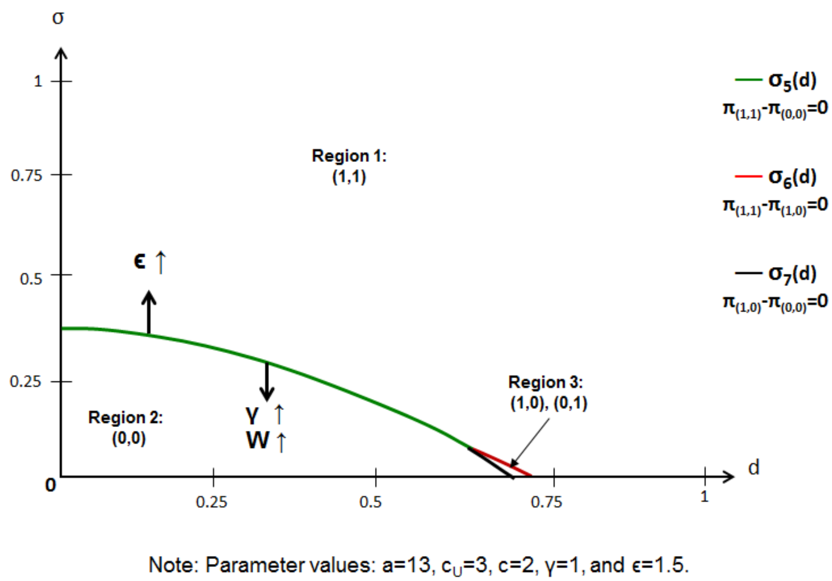


Fig. 3. Innovation equilibria under downstream integration.

Lemma 4 below specifies the conditions under which the different innovation equilibria exist.

Lemma 4 *In R, for given parameters values* $\{a,\ c,\ c_U,\ \gamma,\ \epsilon\}$*, the innovation strategies are characterized as follows:*
i) for $\sigma(d) > \max\{\sigma_5(d), \sigma_6(d)\}$*, the downstream firm innovates in both products* (*Region* 1)*,*
ii) for $\sigma(d) < \min\{\sigma_5(d), \sigma_7(d)\}$*, the downstream firm does not innovate in either product* (*Region* 2)*,*
iii) for $\sigma_7(d) < \sigma(d) < \sigma_6(d)$*, the downstream firm innovates in one product* (*Region* 3)*.*
We have $d\sigma_i/dW < 0$*,* $d\sigma_i/d\gamma < 0$*, and* $d\sigma_i/d\epsilon > 0$*, with* $i = 5, 6, 7$*.*

The effect of d on innovation remains positive (see Lemma 2). The difference with respect to the Base Case, is that the effect of σ is now positive. The reason is that innovation occurs as a result of the internalization of the spillovers when a downstream merger takes place. The downstream firm engages in innovation in both products when the knowledge acquired during the innovation process in one product is highly applicable to the other one. Thus, the presence of a high level of adaptability or applicability between innovation projects becomes innovation-enhancing. The spillovers' internalization reinforces the unit cost reduction associated with innovation. As a consequence, for high values of d and σ, the equilibrium in innovation strategies is now $(1,1)$, whereas it was $(0,0)$ in the Base Case. The proposition that follows arises from the comparison of the results in Lemmas 2 and 4.

Proposition 2 *Downstream integration has a positive effect on innovation.*

Thus, the downstream firm has more incentives to innovate in the presence of a downstream monopoly. This is explained by the fact that the wholesale prices in scenario B are smaller than those in scenario 0, i.e.,

$p^B_{wi}(x_i, x_j) < p^0_{wi}(x_i, x_j)$, and the unpleasant effect of innovation on the wholesale prices is now smoother.[13] The comparative static effects of parameters W, γ, and ϵ have the same interpretation as in the Base Case, i.e., an increase in W or γ, enlarges the region where innovation occurs and an increase in ϵ shrinks it.

At this point, we have analysed the effects on innovation produced by horizontal mergers at either the upstream or the downstream level. In the next section, we assess the effects of the market structure on consumer welfare and the policy implications regarding innovation.

4 The Effect of Horizontal Mergers on Welfare

We compare the consumer surplus under all the scenarios considered, since 'a first objective upon which merger analysis may be based is the protection of consumer interests' (Röller et al., 2000). Competition and antitrust authorities use this criterion to assess the welfare effects of horizontal mergers. With linear demand functions, this is tantamount to comparing quantities. As pointed out by Banal-Estañol et al. (2008), 'this is consistent with the current standards used both in the US and the EU to assess mergers. In the US, the '*substantial lessening of competition*' (SLC) test has been interpreted such that a merger is unlawful if it is likely that it will lead to an increase in price (i.e., to a decrease in consumer surplus). In the EU, the Horizontal Merger Guidelines state that the Commission should take into account, above all, the interests of consumers when considering efficiency claims of merging firms (art. 79-81)'. Subsequent papers, such as Duso et al. (2014) and Flores-Fillol et al. (2014), have also used this criterion.

Before comparing consumer welfare across scenarios, our model confirms that innovation is beneficial for consumers for a given scenario (0, T, and B), as shown in the lemma below.

Lemma 5 *Considering all the possible innovation strategies that can be chosen by downstream firms in each scenario,* $q^\kappa(1,1) > q^\kappa(1,0) = q^\kappa(0,1) > q^\kappa(0,0)$, *for* $\kappa \in \{0, T, B\}$.

Based on a pairwise comparison of equilibrium total output ($q = q_i + q_j$) under Upstream and Downstream Integration (see equation (17)) with respect to the Base Case (see equation (10)), the following proposition arises.

Proposition 3 *For a given innovation strategy across scenarios,*
i) $q^0(1,1) > q^T(1,1) = q^B(1,1)$,
ii) $q^0(0,0) > q^T(0,0) = q^B(0,0)$, *and*
iii) $q^0(1,0) > q^T(1,0) = q^B(1,0)$.
Therefore, consumer welfare is maximized in scenario 0, *and it is the same under scenarios T and B.*

In the light of the previous proposition, we find that consumer welfare is maximized in the absence of mergers when merger formation does not affect the innovation strategy chosen by downstream firms. Thus, competition upstream and downstream enhances consumer welfare in a two-tier market structure with two differentiated outputs. However, as we have seen in Section 2, merger formation affects the equilibrium in innovation strategies. For example, innovation requires products to be highly differentiated and a low level of spillovers under Upstream Integration. By contrast, under Downstream Integration, innovation requires

[13] It can be checked that $\frac{\partial p^B_{wi}}{\partial c_i}\frac{\partial c_i}{\partial x_i} < \frac{\partial p^0_{wi}}{\partial c_i}\frac{\partial c_i}{\partial x_i}$, where $\frac{\partial p^0_{wi}}{\partial c_i}\frac{\partial c_i}{\partial x_i} = \frac{\gamma}{4}\left(2 + \frac{d}{4+d-2d^2} - \frac{d}{4-d-2d^2}\right)$ and $\frac{\partial p^B_{wi}}{\partial c_i}\frac{\partial c_i}{\partial x_i} = \frac{\gamma(2-d^2)}{(4-d^2)}$.

products to be close substitutes and a high level of spillovers (see Figs. 2 and 3). Therefore, to perform a comprehensive welfare analysis, what happens to consumer welfare when the equilibrium R&D effort changes as a consequence of horizontal mergers remains to be analysed.

First, let us compare scenario 0 and scenario T (see jointly Figs. 1 and 2). Thus, if we start from a particular point (d, σ) in scenario 0 such tat the innovation equilibrium strategy is $(1,1)$, it is relevant to compare the total output at this point, i.e., $q^0(1,1)$, with respect to the total output corresponding to the other innovation strategies that can occur at this point under scenario T: either $q^T(0,0)$ or $q^T\{(1,0),(0,1)\}$.[14] Now, if we consider another particular point in scenario 0, at which the innovation strategy is $\{(1,0),(0,1)\}$, it is relevant to compare the total output at this point with the total output corresponding to the innovation strategy that occurs at this point under scenario T, which is $q^T(0,0)$. Then, we obtain the following result.

Proposition 4 *Suppose that the innovation strategies are not the same in scenarios* 0 *and* T. *Then*
i) $q^0(1,1) > q^T(0,0)$,
ii) $q^0(1,1) > q^T\{(1,0),(0,1)\}$,
iii) $q^0\{(1,0),(0,1)\} > q^T(0,0)$,
and, therefore, upstream horizontal mergers are detrimental to innovation and consumer welfare.

Proposition 4 confirms that upstream integration undermines innovation and consumer welfare, because the upstream monopoly under scenario T sets the greatest wholesale prices of all the scenarios considered. As a result of the horizontal integration between upstream firms, the region where both firms innovate is reduced. More precisely, a share of Region 1 in scenario 0 (Fig. 1), where both downstream firms innovate, falls into Regions 3 and 4 in scenario T (Fig. 2), i.e., regions where either just one downstream firm innovates or neither of them engages in innovation activities.

Now we consider Figs 1 and 3 corresponding to scenarios 0 and B. We observe that the following comparisons become relevant: $q^0\{(1,0),(0,1)\}$ versus $q^B(1,1)$, $q^0(0,0)$ versus $q^B(1,1)$, and $q^0(0,0)$ versus $q^B\{(1,0),(0,1)\}$. Let us define $W_1^\dagger \equiv \frac{\gamma(2+d)(1-d)(1+\sigma)}{d}$ and $W_2^\dagger \equiv \frac{\gamma\left(4-d-2d^2\right)(1+\sigma)}{d}$. Then, the following proposition arises.

Proposition 5 *When* $W \in \left(\underline{W}, W_1^\dagger\right)$ *and firms adopt different innovation strategies in equilibrium under scenarios* 0 *and* B, *we have*
i) $q^B(1,1) > q^0\{(1,0),(0,1)\}$,
ii) $q^B(1,1) > q^0(0,0)$,
iii) $q^B\{(1,0),(0,1)\} > q^0(0,0)$,
and, therefore, downstream horizontal mergers are innovation and consumer welfare enhancing.
For $W > \max\left\{W_2^\dagger, \underline{W}\right\}$, *the opposite results are obtained.*

Comparing scenario 0 with scenario B, we observe that consumer welfare depends on W. When the markets are relatively small (i.e., for $W \in (\underline{W}, W_1^\dagger)$) and a downstream merger implies a change in the equilibrium R&D effort, consumer welfare is larger under downstream integration than under the base case. Innovation enhances consumer welfare (see Lemma 5). In most cases, horizontal mergers favour innovation but are detrimental to consumer welfare, i.e., there is a trade-off between promoting innovation in the long run and increasing consumer welfare in the short run. However, Proposition 5 above shows that there is no such trade-off when

[14] Note that the comparison between $q^0(1,1)$ and $q^T(1,1)$ has already been made in Proposition 3.

the markets are sufficiently small and, in that case, downstream horizontal mergers can be both innovation and consumer welfare enhancing in the short run.[15]

5 Policy implications and concluding remarks

This paper explores the effect of horizontal mergers in a vertically-related industry with two upstream and two downstream firms, which can undertake innovation activities. We first conclude that upstream integration (downstream integration) has a negative (positive) effect on innovation. Regarding consumer welfare, we show that downstream mergers can be beneficial.

The policy implications of this paper are as follows. Horizontal mergers between upstream firms should always be forbidden because they are detrimental to innovation and consumer welfare. On the other hand, downstream horizontal mergers can be innovation-enhancing and, therefore, they may have a short run negative effect on consumer welfare (in large markets) and a long-run positive effect derived from innovation. Furthermore, downstream horizontal mergers can even be beneficial for consumers in the short run when the markets are sufficiently small.

As a consequence, competition authorities should distinguish between upstream and downstream horizontal mergers and, in the case of downstream mergers, assess their short run and long-run effects.

The results in this paper can be generalized in different directions. First, enlarging the number of firms should not change our results substantially since we consider a duopoly setting with price competition and product differentiation, in which the prices converge towards the marginal costs (i.e., perfect competition) as the degree of product differentiation decreases. Second, a multiple-sourcing relationship between upstream and downstream firms would increase the competition at the upstream level and, therefore, increase the negative effect of upstream mergers (this extension would connect our model with the literature on bundling). However, the presence of product differentiation among the upstream firms would downplay this negative effect.

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A Appendix. Positivity, Second-Order, and Stability Conditions

In this appendix, we elucidate the conditions that ensure positive quantities and compliance with second-order and stability conditions in all the scenarios considered.

A.1 Positivity conditions

Claim 1 *All the prices and quantities under the Base Case, Upstream Integration, and Downstream Integration are positive for* $W > \underline{W} \equiv \frac{\gamma(d-\sigma)}{1-d}$.

The threshold value $\underline{W}$ is obtained as $\underline{W} = \max\{W_1, ..., W_8\}$, where the different W_i for $i = 1, ..., 8$ come from the following positivity conditions.

♦ *Base Case*

In Stage 2, a necessary condition for equation (8) to be positive is

$$W > W_1 \equiv \gamma \frac{2d - d^3 + \sigma\left(9d^2 - 8 - 2d^4\right) - c_U\left(4 - d - 2d^2\right)\left(4 + d - 2d^2\right)}{(1-d)(2+d)(4+d-2d^2)}.$$

In Stage 3, the price and output in equations (9) and (10) are positive when

$$\begin{aligned} W &> W_2 \equiv \frac{\gamma\left(2-d^2\right)(1+\sigma) - (c+c_U)(2-d)\left(4-d-2d^2\right)}{2(1-d)(3-d^2)}, \\ W &> W_3 \equiv \gamma \frac{2d - d^3 - 8\sigma + 9d^2\sigma - 2d^4\sigma}{(1-d)(2+d)(4+d-2d^2)}. \end{aligned}$$

♦ *Upstream Integration*

In Stage 3, the price and output in equations (16) and (17) are positive when

$$\begin{aligned} W &> W_4 \equiv \frac{\gamma(2+d+2\sigma+d\sigma) - 2(c+c_U)(2-d)(2+d)}{(2+d)(3-2d)}, \\ W &> W_5 \equiv \gamma \frac{d - 2\sigma + d^2\sigma}{(1-d)(2+d)}. \end{aligned}$$

♦ *Downstream Integration*

In Stage 2, the wholesale price in equation (23) is positive when

$$W > W_6 \equiv \frac{\gamma\left(d - 2\sigma + d^2\sigma\right) - c_U(2-d)(2+d)}{(1-d)(2+d)}. \blacksquare$$

A.2 Second-Order Conditions

♦ *Base Case*

From equation (2), in Stage 3, we obtain

$$\frac{\partial^2 \pi_{Di}}{\partial p_i^2} = -\frac{2}{1-d^2} < 0.$$

From equation (3), in Stage 2, we obtain

$$\frac{\partial^2 \pi_{U_i}}{\partial p_{wi}^2} = -\frac{2\left(2-d^2\right)}{(4-d^2)(1-d^2)} < 0.$$

♦ *Upstream Integration*

From equation (2), in Stage 3, we obtain

$$\frac{\partial^2 \pi_{Di}^T}{\partial p_i^2} = -\frac{2}{1-d^2} < 0.$$

From equation (19), in Stage 2, we obtain

$$\frac{\partial^2 \pi_U^T}{\partial p_{wi}^2} = -\frac{2\left(2-d^2\right)}{\left(4-d^2\right)\left(1-d^2\right)} < 0,$$

and

$$\begin{aligned}\frac{\partial^2 \pi_U^T}{\partial p_{wi}^2}\frac{\partial^2 \pi_U^T}{\partial p_{wj}^2} - \left(\frac{\partial^2 \pi_U^T}{\partial p_{wi}\partial p_{wj}}\right)^2 &= \frac{4\left(2-d^2\right)^2}{\left(4-d^2\right)^2\left(1-d^2\right)^2} - \frac{4d^2}{\left(4-d^2\right)^2\left(1-d^2\right)^2} \\ &= \frac{4}{\left(4-d^2\right)\left(1-d^2\right)} > 0.\end{aligned}$$

♦ *Downstream Integration*

From equation (20), in Stage 3, we obtain

$$\frac{\partial^2 \pi_D^B}{\partial p_i^2} = -\frac{2}{1-d^2} < 0,$$

and

$$\frac{\partial^2 \pi_D^B}{\partial p_i^2}\frac{\partial^2 \pi_D^B}{\partial p_j^2} - \left(\frac{\partial^2 \pi_D^B}{\partial p_i\partial p_j}\right)^2 = \frac{4}{\left(1-d^2\right)^2} - \frac{4d^2}{\left(1-d^2\right)^2} = \frac{4}{1-d^2} > 0.$$

From equation (3), in Stage 2, we obtain

$$\frac{\partial^2 \pi_{Ui}^B}{\partial p_{wi}^2} = -\frac{1}{1-d^2} < 0. \blacksquare$$

A.3 Stability Conditions

The Nash equilibria in Stages 2 and 3 need to comply with the stability conditions (see chapter 2 in Vives (2001) for further details).

♦ *Base Case*

The Stage-3 Nash equilibrium prices $p_i^0(x_i, x_j)$ and $p_j^0(x_i, x_j)$ are stable for

$$\left|\frac{\partial^2 \pi_i^0}{\partial p_i^2}\frac{\partial^2 \pi_j^0}{\partial p_j^2}\right| > \left|\frac{\partial^2 \pi_i^0}{\partial p_i\partial p_j}\frac{\partial^2 \pi_j^0}{\partial p_i\partial p_j}\right| \implies \frac{4}{\left(1-d^2\right)^2} > \frac{d^2}{\left(1-d^2\right)^2},$$

which is always observed.

The Stage-2 Nash equilibrium wholesale prices $p_{wi}^0(x_i, x_j)$ and $p_{wj}^0(x_i, x_j)$ are stable for

$$\begin{aligned}\left|\frac{\partial^2 \pi_{Ui}^0}{\partial p_{wi}^2}\frac{\partial^2 \pi_{Uj}^0}{\partial p_{wj}^2}\right| &> \left|\frac{\partial^2 \pi_{Ui}^0}{\partial p_{wi}\partial p_{wj}}\frac{\partial^2 \pi_{Uj}^0}{\partial p_{wi}\partial p_{wj}}\right| \implies \\ \frac{4\left(2-d^2\right)^2}{\left(4-d^2\right)^2\left(1-d^2\right)^2} &> \frac{d^2}{\left(4-d^2\right)^2\left(1-d^2\right)^2},\end{aligned}$$

which is always observed.

♦ *Upstream Integration*

In Stage 3, the stability condition is the same as in the Base Case.

♦ *Downstream Integration*

The Stage-2 Nash equilibrium wholesale prices $p_{wi}^{B}(x_i, x_j)$ and $p_{wj}^{B}(x_i, x_j)$ are stable for

$$\left| \frac{\partial^2 \pi_{Ui}^B}{\partial p_{wi}^2} \frac{\partial^2 \pi_{Uj}^B}{\partial p_{wj}^2} \right| > \left| \frac{\partial^2 \pi_{Ui}^B}{\partial p_{wi} \partial p_{wj}} \frac{\partial^2 \pi_{Uj}^B}{\partial p_{wi} \partial p_{wj}} \right| \implies \frac{1}{(1-d^2)^2} > \frac{d^2}{4(1-d^2)^2},$$

which is always observed. ■

B Appendix. Proofs of Lemmas and Propositions

B.1 Proof of Lemma 1

First, notice that $\frac{\partial p_{wi}^0}{\partial x_i} > 0$ is tantamount to $\frac{\partial p_{wi}^0}{\partial c_i} \frac{\partial c_i}{\partial x_i} > 0$, which is always observed since $\frac{\partial p_{wi}^0}{\partial c_i} = -\frac{1}{4}\left(2 - \frac{d}{4-d-2d^2} + \frac{d}{4+d-2d^2}\right) < 0$ and $\frac{\partial c_i}{\partial x_i} = -\gamma < 0$.

Second, $\frac{\partial p_{wj}^0}{\partial x_i} > 0$ is tantamount to $\frac{\partial p_{wj}^0}{\partial c_j} \frac{\partial c_j}{\partial x_i} > 0$, which is always observed since $\frac{\partial p_{wj}^0}{\partial c_j} = -\frac{1}{4}\left(2 - \frac{d}{4-d-2d^2} + \frac{d}{4+d-2d^2}\right) < 0$ and $\frac{\partial c_j}{\partial x_i} = -\sigma\gamma < 0$.

Finally, $\frac{\partial^2 p_{wi}^0}{\partial x_i \partial d} = -\frac{\gamma}{4}\left[\frac{d(1+4d)}{4-d-2d^2} + \frac{d(1-4d)}{4+d-2d^2} + \frac{1}{4-d-2d^2} + \frac{1}{4+d-2d^2}\right] < 0$ and $\frac{\partial^2 p_{wj}^0}{\partial x_i \partial d} = -\frac{\sigma\gamma}{4}\left[\frac{d(1+4d)}{4-d-2d^2} + \frac{d(1-4d)}{4+d-2d^2} + \frac{1}{4-d-2d^2} + \frac{1}{4+d-2d^2}\right] < 0$. ■

B.2 Proof of Lemma 2

Define $\sigma_1(d)$ implicitly by $F \equiv \pi_{Di}^0(1,0) - \pi_{Di}^0(0,0) = 0$, where $\pi_{Di}^0(1,0)$ and $\pi_{Di}^0(0,0)$ are obtained from equation (11). It follows that $\pi_{Di}^0(1,0) > \pi_{Di}^0(0,0)$ for $\sigma(d) < \sigma_1(d)$ and $\pi_{Di}^0(1,0) < \pi_{Di}^0(0,0)$ for $\sigma(d) > \sigma_1(d)$, where $\sigma_1(d)$ is given by

$$\sigma_1(d) = \frac{\gamma d\left(2-d^2\right)^3\left[W(1-d)(2+d)\left(4+d-2d^2\right) + \gamma\left(8-9d^2+2d^4\right)\right] - \sqrt{A}}{\gamma^2 d^2 \left(2-d^2\right)^4}, \tag{26}$$

with $A = \gamma^2 d^2 (1-d)(2+d)^2\left(2-d^2\right)^4\left(4+d-2d^2\right)^2\left[W^2(1-d)\left(2-d^2\right)^2 + \epsilon(2-d)^2(1+d)\left(4-d-2d^2\right)^2\right]$.

Similarly, define $\sigma_2(d)$ implicitly by $G \equiv \pi_{Di}^0(1,1) - \pi_{Di}^0(0,1) = 0$, where $\pi_{Di}^0(1,1)$ and $\pi_{Di}^0(0,1)$ are obtained from equation (11). It follows that $\pi_{Di}^0(1,1) > \pi_{Di}^0(0,1)$ for $\sigma(d) < \sigma_2(d)$ and $\pi_{Di}^0(1,1) < \pi_{Di}^0(0,1)$ for $\sigma(d) > \sigma_2(d)$, where $\sigma_2(d)$ is given by

$$\begin{aligned}\sigma_2(d) &= \frac{\gamma d(W+\gamma)\left(2-d^2\right)\left(8-9d^2+2d^4\right) - \sqrt[2]{B}(2+d)^2\left(1-d^2\right)\left(4+d-2d^2\right)^2\left(4-d-2d^2\right)^2}{\gamma^2 d\left(2-d^2\right)\left(16-2d-18d^2+d^3+4d^4\right)} \\ &\quad - \frac{\left[Wd^2\left(2-d^2\right)^2 - \gamma\left(64-140d^2+109d^4-35d^6+4d^8\right)\right]}{\gamma d\left(2-d^2\right)\left(16-2d-18d^2+d^3+4d^4\right)},\end{aligned} \tag{27}$$

with

$$B = \gamma^2\left(2-d^2\right)^3 \frac{Wd(1-d)\left(2-d^2\right)^2\left[Wd\left(2-d^2\right)+2\gamma(2-d)(1+d)\left(4-d-2d^2\right)\right] + \left(4-d-2d^2\right)^2(2-d)^2(1+d)\left[\gamma^2\left(1-d^2\right)\left(2-d^2\right) - d\epsilon\left(16-2d-18d^2+d^3+4d^4\right)\right]}{(2-d)^4(1-d)(1+d)^2(2+d)^2\left(4-d-2d^2\right)^4\left(4+d-2d^2\right)^2}.$$

From the properties of $\sigma_1(d)$ and $\sigma_2(d)$, the different regions of equilibria arise straightforwardly. Regarding the partial derivatives from F we obtain:[16]

$$\frac{d\sigma_1}{dW} = -\frac{\partial F/\partial W}{\partial F/\partial \sigma} > 0,\ \frac{d\sigma_1}{d\gamma} = -\frac{\partial F/\partial \gamma}{\partial F/\partial \sigma} > 0,\ \frac{d\sigma_1}{d\epsilon} = -\frac{\partial F/\partial \epsilon}{\partial F/\partial \sigma} < 0,$$

since

$$\begin{aligned}
\frac{\partial F}{\partial W} &= \frac{2\left(2-d^2\right)\left(1-d^2\right)}{(2-d)(1+d)(4-d-2d^2)}\left[q_i^0(1,0) - q_i^0(0,0)\right] > 0,\\
\frac{\partial F}{\partial \gamma} &= 2\frac{\left(2-d^2\right)\left(8-9d^2+2d^4-2d\sigma+d^3\sigma\right)}{(4-d^2)(4-d-2d^2)(4+d-2d^2)} q_i^0(1,0) > 0,\\
\frac{\partial F}{\partial \epsilon} &= -1 < 0,\\
\frac{\partial F}{\partial \sigma} &= -\frac{2d\left(2-d^2\right)^2}{(4-d^2)(4-d-2d^2)(4+d-2d^2)} q_i^0(1,0) < 0.
\end{aligned}$$

Regarding the partial derivatives from G, we obtain

$$\frac{d\sigma_2}{dW} = -\frac{\partial G/\partial W}{\partial G/\partial \sigma} > 0,\ \frac{d\sigma_2}{d\gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial \sigma} > 0,\ \frac{d\sigma_2}{d\epsilon} = -\frac{\partial G/\partial \epsilon}{\partial G/\partial \sigma} < 0,$$

since

$$\begin{aligned}
\frac{\partial G}{\partial W} &= \frac{2\left(2-d^2\right)\left(1-d^2\right)}{(2-d)(1+d)(4-d-2d^2)}\left[q_i^0(1,1) - q_i^0(0,1)\right] > 0,\\
\frac{\partial G}{\partial \gamma} &= \frac{2\left(2-d^2\right)\left(1-d^2\right)}{(2-d)(1+d)(4-d-2d^2)}\left[q_i^0(1,1) - q_i^0(0,1)\right] > 0,\\
\frac{\partial G}{\partial \epsilon} &= -1 < 0,\\
\frac{\partial G}{\partial \sigma} &= \frac{2\gamma\left(1-d^2\right)\left(2-d^2\right)}{(2-d)(1+d)(4-d-2d^2)}\left[q_i^0(1,1) - q_i^0(0,1)\frac{8-9d^2+2d^4}{(1-d)(2+d)(4+d-2d^2)}\right] < 0, \text{ for } W > W^*,
\end{aligned}$$

where $W^* = \frac{\gamma\left[64-16d-140d^2+26d^3+109d^4-13d^5-35d^6+2d^7+4d^8-\sigma d\left(2-d^2\right)\left(16-2d-18d^2+d^3+4d^4\right)\right]}{d(1-d)(2+d)(2-d^2)(4+d-2d^2)}$. ■

B.3 Proof of Lemma 3

Define $\sigma_3(d)$ implicitly by $H \equiv \pi_{Di}^T(1,0) - \pi_{Di}^T(0,0) = 0$, where $\pi_{Di}^T(1,0)$ and $\pi_{Di}^T(0,0)$ are obtained from equation (18). It follows that $\pi_{Di}^T(1,0) > \pi_{Di}^T(0,0)$ for $\sigma(d) < \sigma_3(d)$, and $\pi_{Di}^T(1,0) < \pi_{Di}^T(0,0)$ for $\sigma(d) > \sigma_3(d)$, where $\sigma_3(d)$ is given by

$$\sigma_3(d) = \frac{1}{\gamma d}\left\{W(1-d)(2+d) + \gamma\left(2-d^2\right) - \frac{\sqrt{\gamma^2 d^2 (1-d)(2+d)^2\left[W^2(1-d) + 4\epsilon(2-d)^2(1+d)\right]}}{\gamma d}\right\}. \tag{28}$$

Similarly, define $\sigma_4(d)$ implicitly by $I \equiv \pi_{Di}^T(1,1) - \pi_{Di}^T(0,1) = 0$, where $\pi_{Di}^T(1,1)$ and $\pi_{Di}^T(0,1)$ are obtained from equation (18). It follows that $\pi_{Di}^T(1,1) > \pi_{Di}^T(0,1)$ for $\sigma(d) < \sigma_4(d)$, and $\pi_{Di}^T(1,1) < \pi_{Di}^T(0,1)$ for

[16] In the following we make use of the fact that $q_i^0(1,1) > q_i^0(1,0) = q_i^0(0,1) > q_i^0(0,0)$ which is proven in Lemma 5.

$\sigma(d) > \sigma_4(d)$, where $\sigma_4(d)$ is given by

$$\sigma_4(d) = -\frac{\gamma\left[dW(1-d)(2+d)-\gamma\left(4-2d-3d^2+d^3+d^4\right)\right]}{\gamma^2 d\left(4-d-2d^2\right)} - \frac{\sqrt{\gamma^2(-1+d)(2+d)^2\left[(-1+d)\left(dW+\gamma(2-d)(1+d)\right)^2+4d\epsilon(2-d)^2(1+d)\left(4-d-2d^2\right)\right]}}{\gamma^2 d\left(4-d-2d^2\right)}. \tag{29}$$

From the properties of $\sigma_3(d)$ and $\sigma_4(d)$, the different regions of equilibria arise straightforwardly. Regarding the partial derivatives from H we obtain:[17]

$$\frac{d\sigma_3}{dW} = -\frac{\partial H/\partial W}{\partial H/\partial\sigma} > 0, \quad \frac{d\sigma_3}{d\gamma} = -\frac{\partial H/\partial\gamma}{\partial H/\partial\sigma} > 0, \quad \frac{d\sigma_3}{d\epsilon} = -\frac{\partial H/\partial\epsilon}{\partial H/\partial\sigma} < 0,$$

since

$$\begin{aligned}
\frac{\partial H}{\partial W} &= \frac{1-d}{2-d}\left[q_i^T(1,0) - q_i^T(0,0)\right] > 0,\\
\frac{\partial H}{\partial\gamma} &= \frac{2-d^2-\sigma d}{4-d^2}\left[q_i^T(1,0)\right] > 0,\\
\frac{\partial H}{\partial\epsilon} &= -1 < 0,\\
\frac{\partial H}{\partial\sigma} &= -\frac{\gamma d}{4-d^2}\left[q_i^T(1,0)\right] < 0.
\end{aligned}$$

Regarding the partial derivatives from I, we obtain

$$\frac{d\sigma_4}{dW} = -\frac{\partial I/\partial W}{\partial I/\partial\sigma} > 0, \quad \frac{d\sigma_4}{d\gamma} = -\frac{\partial I/\partial\gamma}{\partial I/\partial\sigma} > 0, \quad \frac{d\sigma_4}{d\epsilon} = -\frac{\partial I/\partial\epsilon}{\partial I/\partial\sigma} < 0,$$

since

$$\begin{aligned}
\frac{\partial I}{\partial W} &= \frac{1-d}{2-d}\left[q_i^T(1,1) - q_i^T(0,1)\right] > 0,\\
\frac{\partial I}{\partial\gamma} &= \frac{1-d}{2-d}\left[(1+\sigma)q_i^T(1,1) - \frac{2\sigma-\sigma d^2-d}{(1-d)(2+d)}q_i^T(0,1)\right] > 0, \text{ for } W > W^{\blacklozenge},\\
\frac{\partial I}{\partial\epsilon} &= -1 < 0,\\
\frac{\partial I}{\partial\sigma} &= \frac{\gamma(1-d)}{2-d}\left[q_i^T(1,1) - \frac{2-d^2}{(1-d)(2+d)}q_i^T(0,1)\right] < 0, \text{ for } W > W^{\blacktriangle},
\end{aligned}$$

where $W^{\blacklozenge} = \frac{\gamma\left(-2+2d+d^2-4\sigma+d\sigma+2d^2\sigma\right)}{(1-d)(2+d)}$ and $W^{\blacktriangle} = \frac{\gamma\left(4-2d-3d^2+d^3+d^4-4d\sigma+d^2\sigma+2d^3\sigma\right)}{d(1-d)(2+d)}$.where it is easy to check that $W^{\blacktriangle} > W^{\blacklozenge}$. ■

B.4 Proof of Proposition 1

Straightforward. ■

B.5 Proof of Lemma 4

Define $\sigma_5(d)$ implicitly by $J \equiv \pi_{Di}^B(1,1) - \pi_{Di}^0(0,0) = 0$, where $\pi_{Di}^B(1,1)$ and $\pi_{Di}^B(0,0)$ are obtained from equation (24). It follows that $\pi_{Di}^B(1,1) > \pi_{Di}^B(0,0)$ for $\sigma(d) > \sigma_5(d)$ and $\pi_{Di}^B(1,1) < \pi_{Di}^B(0,0)$ for $\sigma(d) <$

[17] In the following we make use of the fact that $q_i^T(1,1) > q_i^T(1,0) = q_i^T(0,1) > q_i^T(0,0)$, which is proved in Lemma 5.

$\sigma_5(d)$, where $\sigma_5(d)$ is given by

$$\sigma_5(d) \equiv -\frac{1}{\gamma(4-3d^2-2d^3)}\left\{\gamma\left[E+\gamma\left(8-6d^2-d^3\right)\right]- \sqrt{\gamma^2\left[E+\gamma(8-6d^2-d^3)\right]^2+(4-3d^2-2d^3)\left[-\gamma\left(2E+\gamma(4-3d^2-2d^3)\right)+4\epsilon\left(4-d^2\right)^2(1-d^2)\right]}\right\}, \tag{30}$$

with $E \equiv W(1-d)(2+d)^2$.

Similarly, define $\sigma_6(d)$ implicitly by $K \equiv \pi^B_{Di}(1,1) - \pi^B_{Di}(1,0) = 0$, where $\pi^B_{Di}(1,1)$ and $\pi^B_{Di}(1,0)$ are obtained from equation (24). It follows that $\pi^B_{Di}(1,1) > \pi^B_{Di}(1,0)$ for $\sigma(d) > \sigma_6(d)$ and $\pi^B_{Di}(1,1) < \pi^B_{Di}(1,0)$ for $\sigma(d) < \sigma_6(d)$, where $\sigma_6(d)$ is given by

$$\sigma_6(d) = \frac{-\gamma(W+\gamma)+\sqrt{\gamma^2\left[W^2+4\epsilon(2-d)^2(1+d)\right]}}{\gamma^2}. \tag{31}$$

Finally, define $\sigma_7(d)$ implicitly by $L \equiv \pi^B_D(1,0) - \pi^B_D(0,0) = 0$, where $\pi^B_D(1,0)$ and $\pi^B_D(0,0)$ are obtained from equation (24). It follows that $\pi^B_D(1,0) > \pi^B_D(0,0)$ for $\sigma(d) > \sigma_7(d)$ and $\pi^B_D(1,0) < \pi^B_D(0,0)$ for $\sigma(d) < \sigma_7(d)$, where $\sigma_7(d)$ is given by

$$\sigma_7(d) = -\frac{1}{\gamma^2(4-3d^2)}\left\{\gamma\left(E-\gamma d^3\right)- \sqrt{\gamma^2(4-3d^2-d^3)\left[W^2(1-d)(2+d)^2+(4-3d^2+d^3)\left[-2\gamma W-\gamma^2+4\epsilon(4-3d^2)\right]\right]}\right\}. \tag{32}$$

From the properties of $\sigma_5(d)$, $\sigma_6(d)$, and $\sigma_7(d)$, the different regions of equilibria arise straightforwardly. Regarding the partial derivatives from J we obtain:

$$\frac{d\sigma_5}{dW} = -\frac{\partial J/\partial W}{\partial J/\partial\sigma} < 0, \quad \frac{d\sigma_5}{d\gamma} = -\frac{\partial J/\partial\gamma}{\partial J/\partial\sigma} < 0, \quad \frac{d\sigma_5}{d\epsilon} = -\frac{\partial J/\partial\epsilon}{\partial J/\partial\sigma} > 0,$$

since

$$\begin{aligned}\frac{\partial J}{\partial W} &= \frac{\gamma(1+\sigma)}{(2-d)^2(1+d)} > 0,\\ \frac{\partial J}{\partial\gamma} &= \frac{[W+\gamma(1+\sigma)](1+\sigma)}{(2-d)^2(1+d)} > 0,\\ \frac{\partial J}{\partial\epsilon} &= -2 < 0,\\ \frac{\partial J}{\partial\sigma} &= \frac{\gamma[W+\gamma(1+\sigma)]}{(2-d)^2(1+d)} > 0.\end{aligned}$$

Regarding the partial derivatives from K, we obtain

$$\frac{d\sigma_6}{dW} = -\frac{\partial K/\partial W}{\partial K/\partial\sigma} < 0, \quad \frac{d\sigma_6}{d\gamma} = -\frac{\partial K/\partial\gamma}{\partial K/\partial\sigma} < 0, \quad \frac{d\sigma_6}{d\epsilon} = -\frac{\partial K/\partial\epsilon}{\partial K/\partial\sigma} > 0,$$

since

$$\frac{\partial K}{\partial W} = \frac{\gamma(1+\sigma)}{2(2-d)^2(1+d)} > 0,$$
$$\frac{\partial K}{\partial \gamma} = \frac{W(1-d)\left(2+d^2\right)(1+\sigma)+\gamma\left(4-3d^2-2d^3-16\sigma-12d^2\sigma-2d^3\sigma+4\sigma^2-3d^2\sigma^2-2d^3\sigma^2\right)}{2(4-d^2)^2(1-d^2)} > 0, \text{ for } W > W^{\clubsuit},$$
$$\frac{\partial K}{\partial \epsilon} = -1 < 0,$$
$$\frac{\partial K}{\partial \sigma} = \frac{\gamma\left[W(1-d)(2+d)^2+\gamma\left(8-6d^2-d^3+4\sigma-3d^2\sigma-2d^3\sigma\right)\right]}{2(4-d^2)^2(1-d^2)} > 0,$$

where $W^{\clubsuit} = -\frac{\gamma\left(4-3d^2-2d^3-16\sigma-12d^2\sigma-2d^3\sigma+4\sigma^2-3d^2\sigma^2-2d^3\sigma^2\right)}{(1-d)(2+d)^2(1+\sigma)}$.

Finally, regarding the partial derivatives from L, we obtain

$$\frac{d\sigma_7}{dW} = -\frac{\partial L/\partial W}{\partial L/\partial \sigma} < 0, \quad \frac{d\sigma_7}{d\gamma} = -\frac{\partial L/\partial \gamma}{\partial L/\partial \sigma} < 0, \quad \frac{d\sigma_7}{d\epsilon} = -\frac{\partial L/\partial \epsilon}{\partial L/\partial \sigma} > 0,$$

since

$$\frac{\partial L}{\partial W} = \frac{\gamma(1+\sigma)}{2(2-d)^2(1+d)} > 0,$$
$$\frac{\partial L}{\partial \gamma} = \frac{W(1-d)(2+d)^2(1+\sigma)+\gamma\left(4-3d^2-2d^3\sigma+4\sigma^2-3d^2\sigma^2\right)}{2(4-d^2)^2(1-d^2)} > 0,$$
$$\frac{\partial L}{\partial \epsilon} = -1 < 0,$$
$$\frac{\partial L}{\partial \sigma} = \frac{\gamma\left[W(1-d)(2+d)^2+\gamma\left(4\sigma-3d^2\sigma-d^3\right)\right]}{2(4-d^2)^2(1-d^2)} > 0, \text{ for } W > W^{\heartsuit},$$

where $W^{\heartsuit} = \frac{\gamma\left(d^3-4\sigma+3d^2\sigma\right)}{(1-d)(2+d)^2}$ and it is easy to check that $\underline{W} > W^{\heartsuit} > W^{\clubsuit}$. ■

B.6 Proof of Proposition 2

Straightforward. ■

B.7 Proof of Lemma 5

In the Base Case ($\kappa = 0$),

$$\underbrace{\frac{\left(2-d^2\right)[W+\gamma(1+\sigma)]}{(2-d)(1+d)(4-d-2d^2)}}_{q^0(1,1)} > \underbrace{\frac{\left(2-d^2\right)[2W+\gamma(1+\sigma)]}{(2-d)(1+d)(4-d-2d^2)}}_{q^0(1,0)} > \underbrace{\frac{\left(2-d^2\right)2W}{(2-d)(1+d)(4-d-2d^2)}}_{q^0(0,0)}.$$

Under Upstream Integration ($\kappa = T$) and Downstream Integration ($\kappa = B$),

$$\underbrace{\frac{W+\gamma(1+\sigma)}{(2-d)(1+d)}}_{q^T(1,1)=q^B(1,1)} > \underbrace{\frac{W+\frac{\gamma}{2}(1+\sigma)}{(2-d)(1+d)}}_{q^T(1,0)=q^B(1,0)} > \underbrace{\frac{W}{(2-d)(1+d)}}_{q^T(0,0)=q^B(0,0)}. \blacksquare$$

B.8 Proof of Proposition 3

To prove statement (*i*), when both downstream firms innovate, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{W+\gamma(1+\sigma)}{2(2-d)(1+d)}}_{q^0(1,1)} > \underbrace{\frac{W+\gamma(1+\sigma)}{2(2-d)(1+d)}}_{q^T(1,1)=q^B(1,1)}.$$

To prove statement (*ii*), when neither of the two downstream firms innovates, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{W}{(2-d)(1+d)}}_{q^0(0,0)} > \underbrace{\frac{W}{(2-d)(1+d)}}_{q^T(0,0)=q^B(0,0)}.$$

To prove statement (*iii*), when just one downstream firm innovates, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{2W+\gamma(1+\sigma)}{2(2-d)(1+d)}}_{q^0(1,0)} > \underbrace{\frac{2W+\gamma(1+\sigma)}{2(2-d)(1+d)}}_{q^T(1,0)=q^B(1,0)}. \blacksquare$$

B.9 Proof of Proposition 4

To prove statement (*i*), when both downstream firms innovate under the base case and neither firm innovates under upstream integration, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{W+\gamma(1+\sigma)}{(2-d)(1+d)}}_{q^0(1,1)} > \underbrace{\frac{W}{(2-d)(1+d)}}_{q^T(0,0)}.$$

To prove statement (*ii*), when both downstream firms innovate under the base case and one firm innovates under upstream integration, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{W+\gamma(1+\sigma)}{(2-d)(1+d)}}_{q^0(1,1)} > \underbrace{\frac{W+\frac{\gamma}{2}(1+\sigma)}{(2-d)(1+d)}}_{q^T(1,0)}.$$

To prove statement (*iii*), when one firm innovates under the base case and neither firm innovates under upstream integration, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{W+\frac{\gamma}{2}(1+\sigma)}{(2-d)(1+d)}}_{q^0(1,0)} > \underbrace{\frac{W}{(2-d)(1+d)}}_{q^T(0,0)}. \blacksquare$$

B.10 Proof of Proposition 5

To prove statement (*i*), when one firm innovates under the base case and both firms innovate under downstream integration, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{W+\frac{\gamma}{2}(1+\sigma)}{(2-d)(1+d)}}_{q^0(1,0)} < \underbrace{\frac{W+\gamma(1+\sigma)}{(2-d)(1+d)}}_{q^B(1,1)}$$

for $W < W_1^\dagger \equiv \frac{\gamma(2+d)(1-d)(1+\sigma)}{d}$.

To prove statement (*ii*), when neither firm innovates under the base case and both firms innovate under downstream integration, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{W}{(2-d)(1+d)}}_{q^0(0,0)} < \underbrace{\frac{W+\gamma(1+\sigma)}{(2-d)(1+d)}}_{q^B(1,1)}$$

for $W < W_2^\dagger \equiv \frac{\gamma\left(4-d-2d^2\right)(1+\sigma)}{d}$.

To prove statement (*iii*), when neither firm innovates under the base case and one firm innovates under downstream integration, we have

$$\underbrace{\frac{4-2d^2}{4-2d^2-d}\frac{W}{(2-d)(1+d)}}_{q^0(0,0)} < \underbrace{\frac{W+\frac{\gamma}{2}(1+\sigma)}{(2-d)(1+d)}}_{q^B(1,0)}$$

for $W < W_3^\dagger \equiv \frac{\gamma\left(4-d-2d^2\right)(1+\sigma)}{2d}$.

It can be checked that $W_1^\dagger < W_2^\dagger < W_3^\dagger$ and that the interval $\left(\underline{W}, W_1^\dagger\right)$ is non-empty for $\sigma < \frac{d^2+d-2(1-d)-d^3}{2(1-d)+d^3}$, which can occur in all the scenarios that have been studied. For $W > \max\left\{W_2^\dagger, \underline{W}\right\}$ the opposite results are obtained. ■