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On the effect of collusion on domestic and international research joint ventures*

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Abstract

We analyze the social profitability of research joint ventures (RJVs) in an international context when collusion can occur. RJVs can be used as a subterfuge to sustain tacit collusion agreements in the product market, but the effect of collusion is different between domestic and international RJVs. In absence of collusion, both domestic and international RJVs are socially profitable when spillovers are sufficiently large. However, in presence of collusion, international RJVs are socially preferred when internationalization costs are sufficiently high, and both types of RJVs become less profitable when internationalization costs are low. Traditionally, RJVs with collusion are understood to harm consumers and decrease social welfare, and competition policy advises against them on the grounds of their expected negative effects. However, we show that, despite collusion, international RJVs might be welfare improving.

Keywords: collusion; domestic research joint venture; international research joint venture

JEL Classification Numbers: K21, L24, L44, O32

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1 Introduction

Cooperative R&D among enterprises is common practice in all sectors of the economy, and even more in the high-tech sector. These cooperation agreements in the form of research joint ventures (RJVs) enable firms to exploit synergies, share individual risks, internalize R&D spillovers, increase efficiencies, and promote innovation. As a consequence, new products become available and existing products are produced at lower prices which benefits consumers and raises social welfare. For this reason and without much distinction concerning the characteristics of each RJV, regulatory agencies have mainly ruled in favor of these agreements. In this vein, RJVs are typically exempted from restrictive antitrust rules, both in the United States (US) and in the European Union (EU) (Carree *et al.*, 2010; White, 2007). However, there are two reasons that question the common practice when assessing the effects of RJVs. First, there is increasing evidence that cooperation in R&D is used to facilitate collusion in the product market (Duso *et al.*, 2010; Goeree and Helland, 2009; Martin, 1995). Second, with the globalization of the economy, an increasing number of RJVs put together firms located in different countries (Uphoff and Gilman, 2010). Such international RJVs have different effects than domestic RJVs. The objective of this paper is to analyze the effect of RJVs in an international context under the threat that they can be used to reach collusive agreements in the product market.

Current regulatory practice regarding RJVs in the US is based on the Sherman Antitrust Act, embodied in the US Code. Initially, under this code, guidelines were developed to permit or to impose conditions on mergers, as well as to identify and prohibit cartels for its clear detriment of competition. Nowadays, it also serves as the legal framework for regulatory authorities to determine whether or not a joint venture undermines market competition. The 'Report and Recommendations' of the Antitrust Modernization Commission (2007, p. 378) identified over 30 statutory or judicial exemptions (or partial exemptions) from the antitrust laws, among which are cooperative RJVs (White, 2010). In the EU, the legality of joint ventures is also determined by general rules of competition, contained in the EU Competition Law. More precisely, article 101 (3) of *The Treaty on the Functioning of the EU* (2010) facilitates the creation of joint ventures with the aim of fostering technical and economic progress. As in the US, RJVs in the EU are generally exempted from these rules (Gugler and Siebert, 2007).[1]

[1] In the first half of the 1980s, multiple block exemption regulations were issued, including RJVs (Carree *et al.*, 2010). However, over the past two decades, EU antitrust and merger policies placed a greater emphasis on consumer welfare, particularly through a tighter economic analysis.

In the past, the scope of RJVs has only been limited when they have been proved to favor collusive practices in the product market. In these cases, the antitrust legislation procedures have been applied to penalize these anticompetitive practices. In the US, a rule of reason is applied, which requires fact-finders to balance the potential adverse effects and efficiencies, to determine whether its net effect is likely to be beneficial or harmful to consumers (Piriano, 2008).[2] Because of their detrimental competitive effects, a suit was brought against the following RJVs in the US: (*i*) *CITGO Petroleum and Motiva* (in 2006), a RJV between Shell, Texaco, and Saudi Refining, and (*ii*) *Equilon Enterprises* (in 2007), another RJV between Texaco and Shell. However, in both cases the application of the rule of reason led to the dismissal of the suits (Goeree and Helland, 2010). In the EU, in the period 1964-2004, a suit was brought only against two joint ventures (Carree *et al.*, 2010). However, in both cases the agreements were not found to infringe article 101, and neither decision was appealed. Regarding RJVs, to the best of our knowledge, there is no case in which anticompetitive practices were reported.

Current industrial policy tends to favor domestic RJVs as compared to international RJVs. For example, US domestic RJVs are accorded more lenient antitrust treatment by the National Cooperative Research Act (NCRA) to give American firms a cooperative advantage over foreign firms. While some authors defend the creation of "national champions" (Marvel, 1980; Krugman, 1984; Chou, 1986), others defend free competition and equal treatment of domestic and international firms (Ray, 1981; Sakakibara and Porter, 2001; Hollis, 2003). However, the majority of empirical studies support the latter rationale (Clougherty and Zhang, 2008). In this vein, we assess the possibility of giving a different treatment to domestic and international RJVs.

Recent empirical evidence also shows that, in many cases, RJVs are used to as a subterfuge to sustain tacit collusion agreements in the product market. Using US data, Duso *et al.* (2010) show that RJVs involving direct competitors can lead to collusion in the product market. The authors conclude that RJVs have led to a significant reduction in market output in 29% of the cases included in their sample. By contrast, RJVs among non-competitors are found to be welfare enhancing. Also using US data, Goeree and Helland (2010) examine the potential use of RJVs as a vehicle to facilitate collusion. They exploit a recent change in US leniency policy, which aims at making collusive agreements less sustainable, to examine its effects on RJV formation. They find that the number of RJVs drops notably after this policy change, suggesting illegal practices associated to these agreements. On average, the probability of

[2] The rule of reason is applied on a regular basis after the Dagher case in 2005. This rule of reason approach requires an inquiry into all the characteristics of the relevant market.

joining a RJV falls by 34% among telecommunications firms, by 33% among computer and semiconductor manufacturers, and by 27% among petroleum refining firms.

We propose a theoretical model of RJV formation in an international context when collusion can occur. The main novelty of our analysis is to study the effect of international RJVs with collusion. The effect of RJVs and collusion is analyzed in the seminal paper by d'Aspremont and Jacquemin (1988), which shows that RJVs can be welfare enhancing when spillovers are large enough. In a setting without collusion, Suzumura (1992) and Kamien *et al.* (1992) extend the model in d'Aspremont and Jacquemin (1988) to more general forms of R&D cooperation and market structures. Martin (1995) considers tacit collusion in the product market in a Cournot duopoly model where firms can cooperate in R&D, showing that RJVs are used to sustain collusion. This effect can jeopardize the welfare advantage of RJVs. From a different approach, some papers have analyzed RJVs in an international context without collusion.[3] Spencer and Brander (1983) consider government intervention through subsidies and taxes on exports and R&D, concluding that countries do not subsidize R&D when export subsidies are available. Neary and O'Sullivan (1999) analyze the effect of export subsidies in a model where domestic and foreign firms choose R&D either independently or cooperatively and compete in the product market. These subsidies produce different welfare effects depending on the existence of a government commitment to export subsidies.

We analyze the social profitability of research joint ventures (RJVs) in an international context when firms can collude. RJVs can be used as a subterfuge to sustain tacit collusion agreements in the product market, but the effect of collusion is different between domestic and international RJVs. Our analysis is based on a model that extends the work of d'Aspremont and Jacquemin (1988) to the context of international trade. There are two countries with four firms, two in each country. We assume technological spillovers between domestic and foreign firms to be different. Strategic decision making by firms is modeled as a two-stage game. In stage one, firms decide whether or not to form a RJV with another firm, either domestic or foreign. In stage two, firms choose the quantity to produce. Once a RJV is formed, we distinguish two scenarios. Either firms decide non-cooperatively on production levels, or they use the RJV to collude in the production stage. We limit our attention to symmetric outcomes where either two domestic or two international RJVs are formed, along with the base case in which no RJV is formed. Thus, in addition to the base case, we have four different scenarios: (*i*) domestic and (*ii*) international RJVs with no collusion in the production stage, and (*iii*)

[3] Other papers have focused on the effect of RJVs in presence of cost asymmetries (Petit and Tolwinski, 1999) and product differentiation (see Rosenkranz, 1994, and Lambertini *et al.*, 2002) and cost asymmetries.

domestic and (*iv*) foreign RJVs with collusion in the production stage.

Our main findings can be summarized as follows. In absence of collusion, both domestic and international RJVs are socially profitable when spillovers are sufficiently large. The relative magnitude of each spillover effect (domestic and international) determines which of the two types of RJV is more beneficial. In presence of collusion, domestic RJVs are unambiguously welfare reducing whereas international RJVs can be welfare enhancing. While collusion in domestic RJVs yields a *competition-reduction effect*, under international RJVs there is an additional *efficiency-gains effect* since the specialization in domestic markets allows partner firms to save internationalization costs. Therefore, international RJVs become socially profitable when the latter positive effect of collusion dominates the former negative effect. Naturally, when internationalization costs are low, collusion typically decreases socially welfare (both for domestic and international RJVs).

Traditionally, RJVs with collusion are understood to harm consumers and decrease social welfare unambiguously. However, our results introduce a qualification on this statement: international RJVs with collusion might be socially beneficial when internationalization costs are high. Typically, competition policy advises against RJVs that facilitate collusion on the grounds of their expected negative effects. Our results suggest that antitrust authorities should distinguish between domestic and international RJVs and, in certain cases, be more benevolent with international RJVs.

The paper is organized as follows. Section 2 presents the model and the equilibrium (both in production and R&D) in the base case where no research joint ventures (RJVs) are observed. Section 3 analyzes domestic and international RJVs in absence of collusion at the production stage. Section 4 assesses the effect of collusion. Finally, a brief concluding section closes the paper. All the proofs are provided in the Appendix.

2 The model

Consider an industry with four firms located in two countries that produce a homogeneous good. Two firms are located in country A and two firms are located in country B. Each firm decides the quantity to produce for the domestic and for the foreign markets. Each firm i decides the quantity to produce for the domestic market (h_{ij}) and for the foreign market (e_{ij}), with $i = 1, 2$ and $j = A, B$. Thus, the total quantity traded in country j is composed by domestic production and imports, i.e.,

$$q_j = h_j + e_l = h_{1j} + h_{2j} + e_{1l} + e_{2l}, \tag{1}$$

where $j, l = A, B$ and $j \neq l$. Firms face a linear inverse demand function $p_j = a - q_j$ and compete in quantities (à la Cournot).

Firms' production costs are assumed to be linear in total output produced by the firm. Firms can reduce their marginal production costs by undertaking R&D activities, x_{ij}, at cost $\gamma x_{ij}^2/2$ with $\gamma \geq \underline{\gamma} \equiv 9.6$.[4] R&D efforts exerted by a particular firm produce a positive spillover that benefits other firms. These spillovers may have an asymmetric impact in the domestic and the foreign markets. Let us denote by β and $\lambda\beta$ the intensity of spillovers at the domestic and international levels, respectively. Thus, total production cost for firm i in country j is given by

$$CT_{ij} = \left[c - x_{ij} - \beta x_{kj} - \lambda\beta \sum_{i=1,2} x_{il} \right] (h_{ij} + e_{ij}) + \gamma x_{ij}^2/2, \tag{2}$$

where $i, k = 1, 2$ with $i \neq k$ and $a > c > 0$. At this point, it seems sensible to assume $0 \leqslant \lambda \leqslant \overline{\lambda} \equiv (1 - \beta)/2\beta$ so that the own marginal return to R&D effort is larger than the absorbed one. This cost structure builds on the one proposed in d'Aspremont and Jacquemin (1988) adapting it to a framework with international trade.

In addition, selling abroad makes firms incur an additional *internationalization cost,* te_{ij}. This term accounts for learning costs on how to adapt the product to a foreign market, the costs for complying with different legal requirements, higher transportations costs, or the payment of tariffs levied by the foreign country. Thus, profits of a firm i located in country j are given by

$$\pi_{ij} = p_j h_{ij} + p_l e_{ij} - CT_{ij} - te_{ij}. \tag{3}$$

Let us now consider the base case in which firms behave non-cooperatively in both stages of the game, i.e., firms neither engage in RJVs nor they collude in production. In stage 2, firms choose quantities h_{ij} and e_{ij} to maximize profits in Eq. (3). The Cournot-Nash equilibrium values of this stage game (conditional on R&D decisions) are

$$h_{ij}^{02} = \frac{1}{5} \left[a - c + 2t - (1 + \beta - 3\lambda\beta) \sum_{i=1,2,j=A,B} x_{ij} \right] + (1 - \beta\lambda)\, x_{ij} + (1 - \lambda)\, \beta x_{kj} \tag{4}$$

and

$$e_{ij}^{02} = \frac{1}{5} \left[a - c - 3t - (1 + \beta - 3\lambda\beta) \sum_{i=1,2,j=A,B} x_{ij} \right] + (1 - \beta\lambda)\, x_{ij} + (1 - \lambda)\, \beta x_{kj}, \tag{5}$$

[4] This condition ensures compliance with second-order and stability conditions. The proof is in the Appendix.

where superscript 02 denotes stage-2 equilibrium values in the base case. The sole difference between home and foreign production quantities is found in the effect of the internationalization cost, which benefits domestic production. By looking at these expressions along with Eq. (1), we can verify that the existence of internationalization costs reduces total production in both countries. It can also be checked that both h_{ij}^{02} and e_{ij}^{02} increase with x_{ij}, which constitutes a natural firm reaction to a lower marginal production cost.

Plugging these values into Eq. (3), we get the stage-1 profit function that firms maximize through their choices of R&D

$$\pi_{ij} = \left(h_{ij}^{02}\right)^2 + \left(e_{ij}^{02}\right)^2 - \gamma x_{ij}^2/2. \tag{6}$$

The SPNE total quantity is given by

$$q_j^0 = 10\gamma \frac{2(a-c) - t}{25(\gamma - 1) + (2\beta + 4\beta\lambda - 3)^2}, \tag{7}$$

where superscript 0 denotes equilibrium values in the base case. These expressions corroborate the inefficiency associated to the presence of internationalization costs. At this point, we need to impose an upper bound to the marginal internationalization cost to ensure non-negative equilibrium values, which is given by $0 \leqslant t \leqslant \bar{t} \equiv 2(a-c)$.

We compare consumer surplus under all the considered scenarios since competition and antitrust authorities use this criterion to assess the welfare effects of RJVs, mergers, other types of joint ventures, and other agreements among firms. With linear demand functions, this is tantamount to comparing quantities. The comparison of R&D efforts could yield a different ordering than that of quantities, as pointed out by d'Aspremont and Jacquemin (1988). However, our analysis focuses exclusively on the comparison of quantities (and not R&D spending) because competition and antitrust authorities do not take into account the potential (but uncertain) future gains of different R&D efforts when assessing possible anticompetitive practices.

3 RJVs without collusion at the production stage

D'Aspremont and Jacquemin (1988) conclude that (domestic) RJVs without collusion at the production stage are socially profitable for sufficiently large spillover levels. In this section we test this result in a more general context of international competition where both domestic and international RJVs are possible and can have different spillover effects. As mentioned before, research spillovers (synergies, risk sharing, efficiency gains, innovation diffusion, etc.) constitute the main argument for antitrust authorities when assessing RJVs. However, these authorities apparently do not distinguish between domestic and international RJVs, even though the spillovers they generate may be substantially different.

With the base case equilibrium understood, attention now shifts to RJV formation, both at the domestic and international levels. We will consider that firm collaboration on R&D activities does not extend to the realm of production. Since partner firms behave non-cooperatively when choosing their optimal production levels, stage-2 equilibrium values remain the same as in the base case. However, in stage-1 partner firms determine jointly their R&D efforts.

Therefore, in the case of a *domestic RJV*, partner firms solve

$$\max_{x_{1j},x_{2j}} \sum_{i=1,2} \pi_{ij} = \sum_{i=1,2} \left[\left(h_{ij}^{02}\right)^2 + \left(e_{ij}^{02}\right)^2 - \gamma x_{ij}^2/2 \right] \tag{8}$$

and, in the case of an *international RJV*, partner firms solve

$$\max_{x_{iA},x_{iB}} \sum_{j=A,B} \pi_{ij} = \sum_{j=A,B} \left[\left(h_{ij}^{02}\right)^2 + \left(e_{ij}^{02}\right)^2 - \gamma x_{ij}^2/2 \right]. \tag{9}$$

Since the main goal of the paper is to understand the welfare implications of RJVs, in the analysis that follows we will present directly the equilibrium total quantities,[5] which are

$$q_j^D = 10\gamma \frac{2(a-c)-t}{25\gamma - 12 - 4\beta\left[2(3+\lambda) + \beta(1+2\lambda)(3-4\lambda)\right]} \tag{10}$$

and

$$q_j^I = 10\gamma \frac{2(a-c)-t}{25\gamma - 12 - 4\beta\left[1 + 7\lambda - \beta(1+2\lambda)(2-\lambda)\right]}, \tag{11}$$

where superscripts D and I, respectively, denote equilibrium values in the domestic and international RJV cases in absence of collusion. The difference between the two expressions lies in the value of the denominator, which depends on the intensity of spillovers at the domestic and international levels (i.e., β and λ).

From a pairwise comparison of equilibrium quantities under domestic and international RJVs along with the base case where no RJVs are formed, i.e., comparing Eqs. (7), (10), and (11), the following proposition arises.

Proposition 1 *Let $\underline{\gamma} \leqslant \gamma$, $0 \leqslant \lambda \leqslant \overline{\lambda}$, and $0 \leqslant t \leqslant \bar{t}$. When partner firms in a RJV do not collude,*
i) international RJVs are socially preferred for sufficiently high values of $\lambda\beta$,
ii) domestic RJVs are socially preferred for sufficiently high values of β,
iii) no RJV is observed otherwise.

[5] More information on the computations are available from the authors on request.

Quite naturally, each type of RJV requires a minimum level of spillovers' intensity to yield an overall positive effect. The results in Proposition 1 are represented in Fig. 1 below.

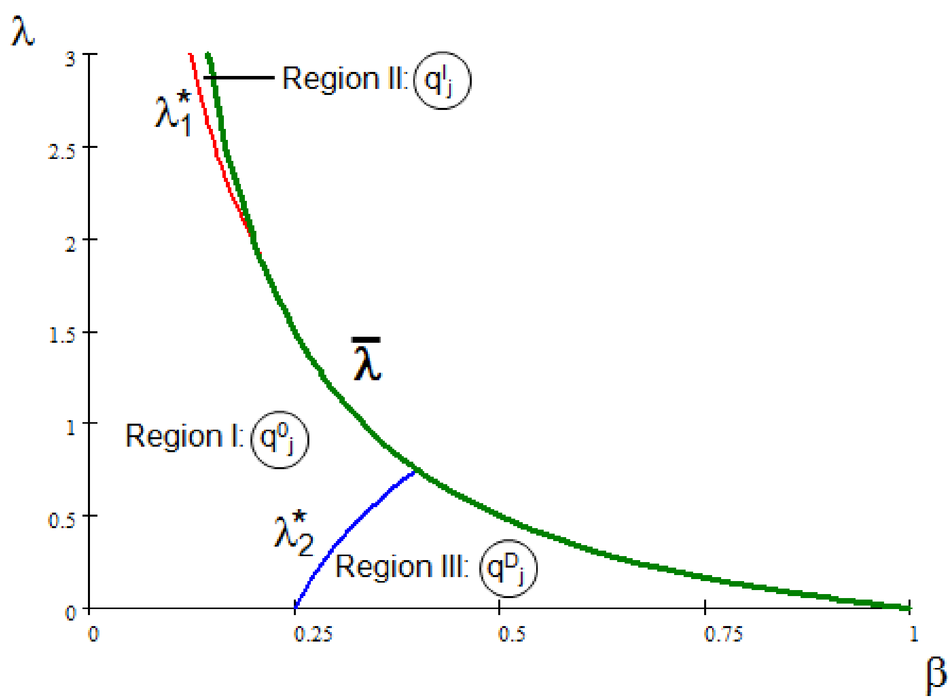

Fig. 1: Socially preferred RJVs without collusion.

Proposition 1(ii) confirms the result in d'Aspremont and Jacquemin (1988) pointing out that (domestic) RJVs are socially preferred when (domestic) spillovers are large enough (which corresponds moving to the east in Fig. 1). Similarly, we find that international RJVs are socially desirable when international spillovers are sufficiently high (which corresponds moving to the north-east in Fig. 1). Moreover, a necessary condition for international RJVs to be more profitable than domestic ones is that international spillovers are larger than domestic ones. ($\lambda > 1$ in Fig. 1). The policy implications of these findings are that each type of RJVs should be allowed if the corresponding spillovers are sufficiently large.

4 RJVs with collusion at the production stage

As mentioned in the introduction, RJVs can be employed as a subterfuge to sustain a tacit collusion agreement at the production stage. Of course, this means that the socially desirability of RJVs is more questionable. In this section, we analyze the consequences of domestic and international RJVs when they involve a collusive behavior. In this case, we assume that partner firms share the market 50/50, so that the RJV behaves as a "merger of equals".[6]

Therefore, in the case of a *domestic RJV*, stage-2 production levels are determined by solving

$$\max_{h_{ij},e_{ij}} \sum_{i=1,2} \pi_{ij} = \sum_{i=1,2} \left[p_j h_{ij} + p_l e_{ij} - CT_{ij} - te_{ij} \right], \tag{12}$$

where $h_{ij} = h_j/2$ and $e_{ij} = e_j/2$. In the case of an *international RJV*, a straightforward efficiency argument suggests partner firms to specialize in their respective domestic markets and avoid exporting to save internationalization costs. As a consequence, $e_{ij} = 0$ and stage-2 production levels are determined by solving

$$\max_{h_{ij}} \sum_{j=A,B} \pi_{ij} = \sum_{j=A,B} \left[p_j h_{ij} - CT_{ij} \right]. \tag{13}$$

Once obtained the results for production,[7] stage-1 partner firms determine jointly their R&D efforts, which yields

$$q_j^{DC} = 3\gamma \frac{2(a-c) - t}{9\gamma - 4 - 4\beta \left[2 + \lambda + \beta (1 + 2\lambda)(1 - \lambda) \right]} \tag{14}$$

and

$$q_j^{IC} = h_j^{IC} = 6\gamma \frac{(a-c)}{9\gamma - 4 - 2\beta \left[1 + 5\lambda - \beta (1 + 2\lambda)(1 - \lambda) \right]}, \tag{15}$$

where superscripts DC and IC denote equilibrium values in the domestic and international RJV cases in presence of collusion, respectively. As in the case without collusion, these equilibrium expressions differ in the intensity of the domestic and international spillovers that affect the denominator of the expressions and, additionally, collusive international RJVs also benefit from being exempt from internationalization costs. Consequently, t does not appear in Eq. (15). From a pairwise comparison of Eqs. (7), (14), and (15), the following proposition arises.

Proposition 2 *Let $\underline{\gamma} \leqslant \gamma$, $0 \leqslant \lambda \leqslant \overline{\lambda}$, and $0 \leqslant t \leqslant \overline{t}$. When partner firms in a RJV collude,*
i) international RJVs are socially preferred for high $t/(a-c)$ and high values of $\lambda\beta$,
ii) domestic RJVs are never socially preferred,
iii) no RJV are preferred for low $t/(a-c)$ and low values of $\lambda\beta$.

[6] It could be argued that concentrating all the production in a single firm could be more efficient. However, capacity constrains and the tacit nature of the collusion agreement between symmetric firms advise in favor of the 50/50 assumption.

[7] More information on the computations are available from the authors on request.

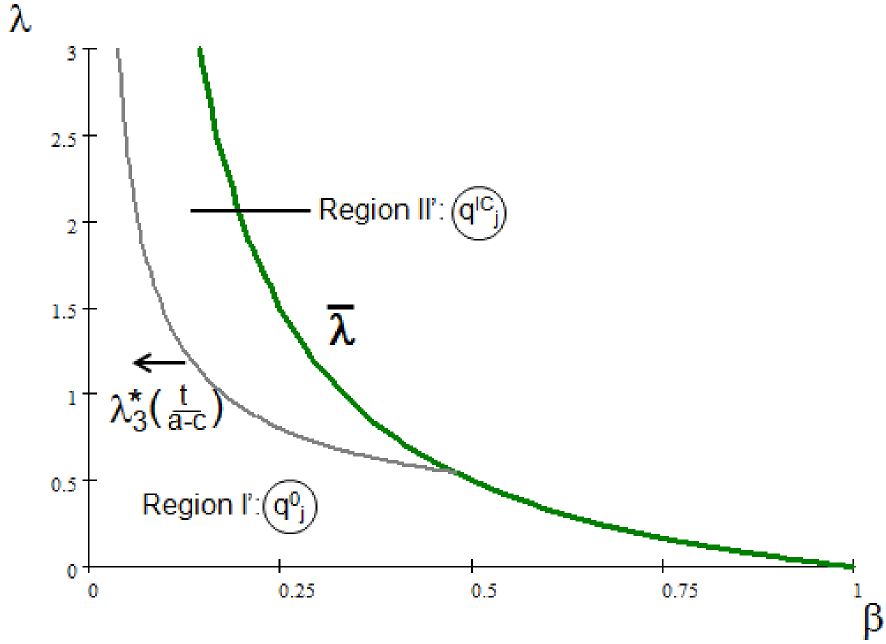

Fig. 2: Socially preferred RJVs with collusion for $\gamma = 10$ and $t/(a-c) = 4/11$.

Arrows denote the move of functions as $t/(a-c)$ rises.

Comparing Propositions 1 and 2, we find that collusion has a differentiated effect on social welfare under domestic and international RJVs. First, collusion reduces social welfare under domestic RJVs. This *competition-reduction effect* of collusion in RJVs has also been obtained by d'Aspremont and Jacquemin (1988) and Martin (1995) in related models. Thus, region III in Fig. 1 does not show in Fig. 2. Second, under international RJVs, an additional effect of collusion is that is allows partner firms to save internationalization costs since they specialize in domestic markets and do not export (i.e., $e_{ij} = 0$ and $q_j^{IC} = h_j^{IC}$). Thus, firms only absorb spillovers through their domestic production (see Eq. (2)). The higher is the internationalization cost, the larger is this *efficiency-gains effect* of collusion. As a consequence, region II' in Fig. 2 expands (shrinks) as t increases (decreases) and might become larger than region II in Fig 1. However, when internationalization costs are low region II' in Fig. 2 might be smaller than region II in Fig. 1 and even disappear.

5 Policy implications and concluding remarks

The results in this paper can be generalized in different directions. Considering heterogeneous products, the social profitability of international RJVs in the presence of collusion would be somewhat diluted. The reason is that the domestic specialization associated to collusion under international RJVs would also convey a loss of product variety for consumers. Another generalization of the paper would be the extension to different competitive environments: enlarging the larger number of firms would downplay the negative effect of collusion, whereas assuming price competition would exacerbate it.

The policy implications of this paper are as follows. In industries characterized by a low probability of collusion, RJVs (both domestic and international) should be allowed when spillovers are large enough. This recommendation is consistent with the findings in d'Aspremont and Jacquemin (1988). Instead, in industries where RJVs are likely to be used as a subterfuge to sustain a tacit collusion agreement, domestic RJVs should always be forbidden regardless of the intensity of spillovers. By contrast, international RJVs should be allowed in high-spillovers environments as long as efficiency gains stemming from savings on internationalization costs are large enough. This means that international RJVs should be treated more favorably than domestic RJVs under these circumstances.

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A Appendix: Second-order and stability conditions

In this appendix, we elucidate the conditions that ensure positive quantities and compliance with second-order and stability conditions in all the considered scenarios, i.e., we prove the following claim.

Claim 1 *Imposing* $\gamma \geqslant \underline{\gamma} = 9.6$ *is sufficient to ensure compliance with second-order and stability conditions.*

A.1 Second-order conditions

♦ *Base case (no RJVs)*

It can easily checked that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2 \pi_{ij} / \partial x_{ij}^2 < 0$ (see Eq. (6)) we get

$$\gamma > \gamma_1 \equiv \frac{4}{25} \left[4 - \beta \left(1 + 2\lambda\right)\right]^2. \tag{16}$$

A sufficient condition for Eq. (16) to be true, is that $\gamma > \max_{0 \leqslant \lambda \leqslant \overline{\lambda}} \gamma_1 \equiv \gamma_1^{\lambda} = \frac{4}{25} \left(4 - \beta\right)^2$.

♦ *Domestic RJVs without collusion at the production stage*

It can easily checked that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2 \left(\pi_{1j} + \pi_{2j}\right) / \partial x_{1j}^2 < 0$ and $\partial^2 \left(\pi_{1j} + \pi_{2j}\right) / \partial x_{2j}^2 < 0$ (see Eq. (8)) we get

$$\gamma > \gamma_2 \equiv \frac{4}{25} \left[17 + \beta \left(17\beta - 16 - 12\lambda \left(1 + \beta\right) + 8\lambda^2 \beta\right)\right], \tag{17}$$

and positivity of the determinant requires $\left(\gamma_2 - \gamma\right)^2 - \left\{\frac{8}{25} \left[1 + 2\beta \left(\lambda - 2\right)\right] \left[\beta \left(1 + 2\lambda\right) - 4\right]\right\}^2 > 0$, which is observed when

$$\gamma > \gamma_3 \equiv \max \left\{4 \left(\beta - 1\right)^2, \frac{4}{25} \left[\beta \left(4\lambda - 3\right) - 3\right]^2\right\}. \tag{18}$$

A sufficient condition for Eqs. (17) and (18) to be true, is that $\gamma > \max_{0 \leqslant \lambda \leqslant \overline{\lambda}} \gamma_2 \equiv \gamma_2^{\lambda} = \frac{4}{25} \left(17\beta^2 - 16\beta + 17\right)$ and $\gamma > \max_{0 \leqslant \lambda \leqslant \overline{\lambda}} \gamma_3 \equiv \gamma_3^{\lambda} = \max\{4 \left(\beta - 1\right)^2, \frac{36}{25} \left(\beta + 1\right)^2\}$, respectively.

♦ *International RJVs without collusion at the production stage*

It can easily checked that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2 \left(\pi_{iA} + \pi_{iB}\right) / \partial x_{iA}^2 < 0$ and $\partial^2 \left(\pi_{iA} + \pi_{iB}\right) / \partial x_{iB}^2 < 0$ (see Eq. (9)) we get

$$\gamma > \gamma_4 \equiv \frac{4}{25} \left\{17 + \beta \left[\beta \left(2 + \lambda \left(13\lambda - 2\right)\right) - 22\lambda - 6\right]\right\}, \tag{19}$$

and positivity of the determinant requires $(\gamma_4-\gamma)^2-\left\{\frac{8}{25}\left[1-\beta\left(3\lambda-1\right)\right]\left[\beta\left(1+2\lambda\right)-4\right]\right\}^2>0$, which is observed when

$$\gamma>\gamma_5\equiv\max\left\{4\left(\lambda\beta-1\right)^2,\frac{4}{25}\left[\beta\left(\lambda-2\right)+3\right]^2\right\}. \tag{20}$$

A sufficient condition for Eqs. (19) and (20) to be true, is that $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_4\equiv\gamma_4^{\lambda}=\frac{4}{25}\left(2\beta^2-6\beta+17\right)$ and $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_5\equiv\gamma_5^{\lambda}=\max\left\{4,\frac{1}{25}\left(7-5\beta\right)^2\right\}=4$, respectively.

♦ *Domestic RJVs with collusion at the production stage*

It can easily checked that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2\left(\pi_{1j}+\pi_{2j}\right)/\partial x_{1j}^2<0$ and $\partial^2\left(\pi_{1j}+\pi_{2j}\right)/\partial x_{2j}^2<0$ we get

$$\gamma>\gamma_6\equiv\frac{4}{9}\left[\beta\left(\lambda-1\right)-1\right]^2, \tag{21}$$

and positivity of the determinant requires $(\gamma_6-\gamma)^2-\gamma_6^2>0$, which is observed when

$$\gamma>\gamma_7\equiv2\gamma_6. \tag{22}$$

A sufficient condition for Eqs. (21) and (22) to be true, is that $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_6\equiv\gamma_6^{\lambda}\equiv\frac{4}{9}\left(\beta+1\right)^2$ and $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_7\equiv\gamma_7^{\lambda}=2\gamma_6^{\lambda}$, respectively.

♦ *International RJVs with collusion at the production stage*

It can easily checked that second-order conditions at the production stage (stage 1) are always satisfied. At the R&D stage (stage 2), from $\partial^2\left(\pi_{iA}+\pi_{iB}\right)/\partial x_{iA}^2<0$ and $\partial^2\left(\pi_{iA}+\pi_{iB}\right)/\partial x_{iB}^2<0$ we get

$$\gamma>\gamma_8\equiv\frac{1}{9}\left\{8+2\beta\left[\beta\left(1+\lambda^2\right)-4\right]\right\}, \tag{23}$$

and positivity of the determinant requires $(\gamma_8-\gamma)^2-\left[\frac{4}{9}\beta\lambda\left(2-\beta\right)\right]^2>0$, which is observed when

$$\gamma>\gamma_9\equiv\frac{2}{9}\left[\beta\left(\lambda-1\right)+2\right]^2. \tag{24}$$

A sufficient condition for Eqs. (23) and (24) to be true, is that $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_8\equiv\gamma_8^{\lambda}\equiv\frac{1}{18}\left(5\beta^2-18\beta+17\right)$ and $\gamma>\max\limits_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_9\equiv\gamma_9^{\lambda}\equiv\frac{1}{18}\left(5-3\beta\right)^2$, respectively.

As a result of comparing the previous second-order conditions and using the bounds γ_h^λ for $h = 1, ..., 9$, we compute the lower bound for γ as:[8]

$$\gamma \geqslant \max_{0 \leq \beta \leq 1} \{\gamma_1^\lambda, ..., \gamma_9^\lambda\} = \max_{0 \leq \beta \leq 1} \{4, \frac{36}{25} (\beta + 1)^2\} = 5.76.$$

A.2 Stability conditions

Stability of equilibria is ensured when the Jacobian of first derivatives of profits with respect to R&D investments is negative definite (see chapter 2 in Vives (2001) for further details). This matrix is symmetric with the following structure

$$\begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{pmatrix}.$$

The Jacobian of first derivatives is negative definite if

$$A < 0, \tag{25}$$

$$(A - B)(A + B) > 0, \tag{26}$$

$$2BCD + A\left(A^2 - B^2 - C^2 - D^2\right) < 0, \tag{27}$$

$$\left[(A + B)^2 - (C + D)^2\right]\left[(A - B)^2 - (C - D)^2\right] > 0. \tag{28}$$

Condition in Eq. (25) is already guaranteed by second-order conditions.

Claim 2 *Conditions in Eqs. (26)-(28) are satisfied iff*

$$A - B < 0, \tag{29}$$

$$A + B < 0, \tag{30}$$

$$(A + B)^2 - (C + D)^2 > 0, \tag{31}$$

$$(A - B)^2 - (C - D)^2 > 0. \tag{32}$$

Proof. First, notice that Eqs. (29) and (30) guarantee that (26) holds and Eqs. (31) and (32) guarantee that (28) holds. Finally, Eq. (27) can be rewritten as:

$$(A - B)^2 \left(2A(A + B) - (C + D)^2\right) > (C - D)^2 (A - B)(A + B). \tag{33}$$

[8] It can be checked that $\gamma_1^\lambda < \gamma_5^\lambda$, $\gamma_2^\lambda < \gamma_5^\lambda$, $\gamma_4^\lambda < \gamma_5^\lambda$, $\gamma_6^\lambda < \gamma_7^\lambda < \gamma_5^\lambda$, and $\gamma_8^\lambda < \gamma_9^\lambda < \gamma_5^\lambda$. Furthermore, the first bound in γ_3^λ is also lower than γ_5^λ, i.e., $4(\beta - 1)^2 < 4$.

Under condition (32), Eq. (33) holds iff

$$2A(A+B)-(C+D)^2 > (A-B)(A+B), \text{ or} \quad (34)$$
$$(A+B)^2-(C+D)^2 > 0, \quad (35)$$

which is Eq. (31). ■

♦ *Base case (no RJVs)*

In this scenario

$$A \equiv \partial^2\pi_{ij}/\partial x_{ij}^2 = \frac{1}{25}\{64-25\gamma+4\beta[1+2\lambda][-8+\beta(1+2\lambda)]\},$$
$$B \equiv \partial^2\pi_{ij}/\partial x_{ij}\partial x_{kj} = \frac{4}{25}[1-2\beta(2-\lambda)][-4+\beta(1+2\lambda)], \text{ and}$$
$$C = D \equiv \partial^2\pi_{ij}/\partial x_{ij}\partial x_{il} = \frac{4}{25}[-4+\beta(1+2\lambda)][1+\beta(1-3\lambda)].$$

Thus, Eq. (32) holds directly and Eqs. (29)-(31) become

$$\gamma > \gamma_{10} \equiv \frac{4}{5}(1-\beta)[4-\beta(1+2\lambda)], \quad (36)$$
$$\gamma > \gamma_{11} \equiv \frac{4}{25}[4-\beta(1+2\lambda)][3+\beta(3-4\lambda)], \quad (37)$$
$$\gamma > \gamma_{12} \equiv \max\left\{\frac{4}{5}[4-\beta(1+2\lambda)][1+\beta(1-2\lambda)], \frac{4}{25}[4-\beta(1+2\lambda)][1+\beta(1+2\lambda)]\right\} \quad (38)$$

A sufficient condition for Eqs. (36)-(38) to be true, is that $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{10} \equiv \gamma_{10}^{\lambda} \equiv \frac{4}{5}(4-\beta)(1-\beta)$, $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{11} \equiv \gamma_{11}^{\lambda} = \frac{12}{25}(4-\beta)(1+\beta)$, and $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}}\gamma_{12} \equiv \gamma_{12}^{\lambda} = \max\left\{\frac{4}{5}(4-\beta)(1+\beta), \frac{24}{25}\right\} = \frac{4}{5}(4-\beta)(1+\beta)$, respectively.

♦ *Domestic RJVs without collusion at the production stage*

In this scenario

$$A \equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{ij}^2 = \frac{1}{25}\left\{68-25\gamma+4\beta\left[-16+17\beta-12\lambda(1+\beta)+8\beta\lambda^2\right]\right\},$$
$$B \equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{1j}\partial x_{2j} = \frac{8}{25}[1-2\beta(2-\lambda)][-4+\beta(1+2\lambda)], \text{ and}$$
$$C = D \equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{1j}\partial x_{il} = \frac{4}{25}[-3+\beta(-3+4\lambda)][1+\beta(1-3\lambda)],$$

for $i = 1, 2$ and $j, l = A, B$. Thus, Eq. (32) holds directly and Eqs. (29)-(31) become

$$\gamma > \gamma_{13} \equiv 4(1-\beta)^2, \quad (39)$$
$$\gamma > \gamma_{14} \equiv \frac{4}{25}[3+\beta(3-4\lambda)]^2, \quad (40)$$
$$\gamma > \gamma_{15} \equiv \max\left\{\frac{4}{5}[3+\beta(3-4\lambda)][1+\beta(1-2\lambda)], \frac{4}{25}[3+\beta(3-4\lambda)][1+\beta(1+2\lambda)]\right\} \quad (41)$$

A sufficient condition for Eqs. (39)-(41) to be true, is that $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{13} \equiv \gamma_{13}^{\lambda} \equiv 4(1-\beta)^2$, $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{14} \equiv \gamma_{14}^{\lambda} = \frac{36}{25}(1+\beta)^2$, and $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{15} \equiv \gamma_{15}^{\lambda} = \max\left\{\frac{12}{5}(1+\beta)^2, \frac{1}{2}(1+\beta)^2\right\} = \frac{12}{5}(1+\beta)^2$, respectively.

♦ *International RJVs without collusion at the production stage*

In this scenario

$$\begin{aligned} A &\equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}^2 = \frac{1}{25}\{68-25\gamma+4\beta[-6-22\lambda+\beta(2+\lambda[13\lambda-2])]\}, \\ B &\equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{iA}\partial x_{iB} = \frac{8}{25}[1+\beta(1-3\lambda)][-4+\beta(1+2\lambda)], \\ C &\equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}\partial x_{kj} = \frac{4}{25}\left[-3+\beta\left(19-3\beta-12\lambda(1+\beta)+13\beta\lambda^2\right)\right], \text{ and} \\ D &\equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}\partial x_{kl} = \frac{4}{25}[1+\beta(1-3\lambda)][-3-\beta(3-4\lambda)], \end{aligned}$$

for $i,k=1,2$, $k\neq i$ and $j,l=A,B$, $l\neq j$. Thus, Eqs. (29)-(32) become

$$\gamma > \gamma_{16} \equiv 4(1-\beta\lambda)^2, \tag{42}$$

$$\gamma > \gamma_{17} \equiv \frac{4}{25}[3-\beta(2-\lambda)]^2, \tag{43}$$

$$\gamma > \gamma_{18} \equiv \max\left\{\frac{4}{5}(1-\beta)[3-\beta(2-\lambda)], \frac{4}{25}[3-\beta(2-\lambda)][1+\beta(1+2\lambda)]\right\}, \tag{44}$$

$$\gamma > \gamma_{19} \equiv \max\{4(1-\beta)(1-\beta\lambda), 4(1-\beta\lambda)[1+\beta(1-2\lambda)]\}. \tag{45}$$

A sufficient condition for Eqs. (42)-(45) to be true, is that $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{16} \equiv \gamma_{16}^{\lambda} \equiv 4$, $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{17} \equiv \gamma_{17}^{\lambda} = \frac{1}{25}(7-5\beta)^2$, $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{18} \equiv \gamma_{18}^{\lambda} = \max\left\{\frac{2}{5}(7-5\beta)(1-\beta), \frac{4}{25}(7-5\beta)\right\}$, and $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{19} \equiv \gamma_{19}^{\lambda} = \max\{4(1-\beta), 4(1+\beta)\} = 4(1+\beta)$, respectively.

♦ *Domestic RJVs with collusion at the production stage*

In this scenario

$$\begin{aligned} A &\equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{ij}^2 = \frac{1}{9}\{4-9\gamma+4\beta[2+\beta(1-\lambda)][1-\lambda]\}, \\ B &\equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{1j}\partial x_{2j} = \frac{4}{9}[1+\beta(1-\lambda)]^2, \text{ and} \\ C &= D \equiv \partial^2(\pi_{1j}+\pi_{2j})/\partial x_{1j}\partial x_{il} = \frac{2}{9}[1+\beta(1-\lambda)][-1+\beta(-1+4\lambda)], \end{aligned}$$

for $i=1,2$ and $j,l=A,B$. Thus, Eq. (32) holds directly and Eqs. (29)-(31) become

$$\gamma > 0, \quad (46)$$
$$\gamma > \gamma_{20} \equiv \frac{8}{9}\left[1+\beta(1-\lambda)\right]^2, \quad (47)$$
$$\gamma > \gamma_{21} \equiv \max\left\{\frac{4}{9}\left[1+\beta(1-\lambda)\right]\left[1+\beta(1+2\lambda)\right], \frac{4}{3}\left[1+\beta(1-\lambda)\right]\left[1+\beta(1-2\lambda)\right]\right\}. \quad (48)$$

Eq. (46) holds by construction. A sufficient condition for Eqs. (47) and (48) to be true, is that $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{20} \equiv \gamma_{20}^{\lambda} \equiv \frac{8}{9}(1+\beta)^2$ and $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{21} \equiv \gamma_{21}^{\lambda} = \max\left\{\frac{1}{2}(1+\beta)^2, \frac{4}{3}(1+\beta)^2\right\} = \frac{4}{3}(1+\beta)^2$, respectively.

♦ *International RJVs with collusion at the production stage*

In this scenario

$$A \equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}^2 = \frac{1}{9}\left\{8-9\gamma-2\beta\left[4-\beta\left(1+\lambda^2\right)\right]\right\},$$
$$B \equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{iA}\partial x_{iB} = \frac{4}{9}\beta\lambda(2-\beta),$$
$$C \equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}\partial x_{kj} = \frac{2}{9}\left\{-2+\beta\left[5+\beta\left(-2+\lambda^2\right)\right]\right\}, \text{ and}$$
$$D \equiv \partial^2(\pi_{iA}+\pi_{iB})/\partial x_{ij}\partial x_{kl} = \frac{2}{9}\beta\lambda(1+\beta),$$

for $i,k=1,2$, $k\neq i$ and $j,l=A,B$, $l\neq j$. Thus, Eqs. (29)-(32) become

$$\gamma > \gamma_{22} \equiv \frac{2}{9}\left[2-\beta(1+\lambda)\right]^2, \quad (49)$$
$$\gamma > \gamma_{23} \equiv \frac{2}{9}\left[2-\beta(1-\lambda)\right]^2, \quad (50)$$
$$\gamma > \gamma_{24} \equiv \max\left\{\frac{2}{3}(1-\beta)\left[2-\beta(1-\lambda)\right], \frac{2}{9}\left[2-\beta(1-\lambda)\right]\left[1+\beta(1+2\lambda)\right]\right\}, \quad (51)$$
$$\gamma > \gamma_{25} \equiv \max\left\{\frac{2}{3}(1-\beta)\left[2-\beta(1+\lambda)\right], \frac{2}{9}\left[2-\beta(1+\lambda)\right]\left[1+\beta(1-2\lambda)\right]\right\}. \quad (52)$$

A sufficient condition for Eqs. (49)-(52) to be true, is that $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{22} \equiv \gamma_{22}^{\lambda} \equiv \frac{2}{9}(2-\beta)^2$, $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{23} \equiv \gamma_{23}^{\lambda} = \frac{1}{18}(5-3\beta)^2$, $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{24} \equiv \gamma_{24}^{\lambda} = \max\left\{\frac{1}{3}(5-3\beta)(1-\beta), \frac{2}{9}(5-3\beta)\right\}$, and $\gamma > \max_{0\leqslant\lambda\leqslant\overline{\lambda}} \gamma_{25} \equiv \gamma_{25}^{\lambda} = \max\left\{\frac{2}{3}(1-\beta)(2-\beta), \frac{2}{9}(1+\beta)(2-\beta)\right\}$, respectively.

As a result of comparing the previous stability conditions and using the bounds γ_h^{λ} for $h = 10, ..., 25$, we compute the lower bound for γ as:[9]

$$\gamma \geqslant \underline{\gamma} \equiv \max_{0\leq\beta\leq1}\{\gamma_{10}^{\lambda}, ..., \gamma_{25}^{\lambda}\} = 9.6.$$

[9] It can be checked that $\gamma_{10}^{\lambda} < \gamma_{12}^{\lambda}$, $\gamma_{11}^{\lambda} < \gamma_{12}^{\lambda} < 4.8$, $\gamma_{13}^{\lambda} < 4$, $\gamma_{14}^{\lambda} < \gamma_{15}^{\lambda} < 9.6$, $\gamma_{16}^{\lambda} < \gamma_{19}^{\lambda}$, $\gamma_{17}^{\lambda} < 1.96$, $\gamma_{18}^{\lambda} < 5.6$, $\gamma_{19}^{\lambda} < 8$, $\gamma_{20}^{\lambda} < \gamma_{21}^{\lambda} < 16/3$, $\gamma_{22}^{\lambda} < 8/9$, $\gamma_{23}^{\lambda} < 25/15$, $\gamma_{24}^{\lambda} < 2$, and $\gamma_{25}^{\lambda} < 4/3$.