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An appraisal of firm size distribution: Does sample size matter?

Agustí Segarra, Mercedes Teruel*

Research Group of Industry and Territory, Department of Economics, Universitat Rovira i Virgili, CREIP, 43204 Reus, Spain

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ABSTRACT

Recent empirical evidence based on extensive databases shows that firm size distributions (FSD) vary with the sample. This paper analyses the effect of sample size on the FSD of Spanish manufacturing firms for the years 2001 and 2006. We use a comprehensive dataset that has two measures of firm size: sales and employment. Our database shows a skewed FSD to the right which there are numerous small firms and a few large firms. Applying a rolling regression to control for sample size developed by Peng (2010), we show the existence of a non-constant power-law distribution that depends on the sampling size. Furthermore, the FSD of employees is more sensitive to firm age than the FSD of sales.

1. Introduction

Many economics facts follow power laws, for instance, the distribution of the populations of cities, the size of firms, the wealth of the richest people and the number of patents all follow a power law. It is common knowledge that the market is made up of a large number of small firms and a small number of large firms and that firm size distributions (FSDs) are highly skewed. For instance, Jovanovic (1982) developed a model of 'noisy' selection where efficient firms grow and survive and inefficient firms decline. This model of selection with incomplete information questioned early studies on the dynamics of industries that found no relation between size and growth rates of firms (Gibrat, 1931), although it is in line with later studies such as Mansfield (1962).

Interest in analyzing FSDs continues to increase, especially because of the important implications such analysis could have for policy. From this evidence, Pavitt et al. (1987) remarked that high variance in FSD within and between sectors makes generalizations difficult, which means that broad policies are likely to be inappropriate. Understanding the power-law distribution is important in economics because a small fraction of firms employ a large number of employees and have significant market power. Consequently, small changes in the distribution of these firms can have a considerable macroeconomic impact.

Here, we review a wide range of empirical evidence and estimate the presence of power-laws in an extensive database of Spanish manufacturing firms in 2001 and 2006. Extensive databases containing more firms, especially small firms, have facilitated new analytical perspectives. Since the late 1990s, the availability of U.S. Census Department data regarding the whole population of U.S. businesses has given rise to new approaches (Axtell, 2001; Teitelbaum and Axtell, 2005; Bottazzi et al., 2008). Recently, a theoretical debate has emerged about whether firm size distributions (FSDs) are best modeled using a power-law distribution or a lognormal distribution. In particular, the main debate is whether firms are distributed as

* Corresponding author. Tel.: +34 977 759 816; fax: +34 977 300 661.
E-mail addresses: agusti.segarra@urv.cat (A. Segarra), mercedes.teruel@urv.cat (M. Teruel).

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power-laws or as lognormal (Aitchinson and Brown, 1954; Champernowne, 1953; Stanley et al., 1995; Urzua, 2000; Axtell, 2001; Mitzenmacher, 2004; de Wit, 2005; Coad, 2009).[1]

This paper aims to improve understanding regarding the dependence of power-laws of FSDs on sample size at firm level. Some studies have pointed out that small samples affect the estimation of the power-law (Stanley et al., 1995). To our knowledge this is the first study that analyzes the effect of the sample size on the estimation of the power-law. We determine the power-law relationship between firm rank and firm size by applying a rolling sample methodology to analyze the impact of including small firms. Furthermore, we provide evidence that firm age can have a varying effect on the estimation of the power-law. In order to do the empirical calibration we use an extensive database for Spanish manufacturing firms that compiles balance sheet information from the Spanish Mercantile Register for the years 2001 and 2006. The firm size distribution and the concentration of industries are associated with the share of fixed and technological capital which differs between industries but which is stable over time. For this reason, we concentrate our empirical work on the first and last year of our database.

Previous literature shows differences between industries in the estimated power-law of the FSD that account for firm size. However, the empirical results are not conclusive, and we therefore believe the diversity of results obtained may reflect differences in sample size. Applying Peng's (2010) methodology, this paper shows how the inclusion of a larger number of firms affects the results reported in the past related literature.[2] We analyze the estimated power-law coefficient according to the sample size. The paper employs a more suitable econometric estimation procedure to account for the shortcomings of the estimation approaches used in the relevant literature. The specific contributions to the existing literature are twofold. First, it analyses two different variables, employees and sales, and reveals some important differences. Second, this paper analyses how firm age affects the power-laws parameters.

The structure of the paper is as follows. Section 2 outlines the theoretical and empirical literature related to firm size distribution and power-laws. Section 3 presents the database belonging to an exhaustive database of Spanish manufacturing firms. Section 4 presents the econometric methodology and provides the results for Spanish manufacturing firms. Section 5 presents the results of the effect of the power-law in terms of firm age, and the final section presents the concluding remarks.

2. Power-laws and firm size distribution

2.1. Previous results

Interest in analyzing FSDs first appeared with Gibrat's Law. In his PhD thesis, Gibrat (1931) observed that the FSD was close to the lognormal distribution, and concluded that firm growth rates follow a random multiplicative process. Several models of proportional growth were subsequently introduced to economics to explain firm growth rates and market dynamics. Those models have become a standard reference point in Industrial Organization when dealing with FSDs.[3]

Since Gibrat's Law, different studies have tried to test the FSD empirically. The first wave of contributions used data that were readily available in public sources (Hart and Prais, 1956; Mansfield, 1962). During the mid-to-late 80s a second wave appeared when access to extensive data became available. These broad data sources provided better coverage of the smallest firms which allowed researchers to analyze full FSD from a dynamic perspective.

Early studies such as Hart and Prais (1956) found evidence that the log normal fits the FSDs reasonably well. However, most recent empirical literature has repeatedly found the lognormal and Pareto distribution to be mixed. According to Crosato and Ganugi (2007), both distributions may be derivable from multiplicative growth models à la Gibrat. Table 1 summarizes some recent empirical studies and their findings.

On the one hand, some studies have found a non-lognormal right tail (Stanley et al., 1995; Hart and Oulton, 1997; Voit, 2001). On the other hand, the Pareto distribution fits the right tail well but not for the entire FSD (Ijiri and Simon, 1977; Steindl, 1965; Okuyama et al., 1999). As a result, recent debate has centered on whether there is variability in the descriptions of the FSD (Bottazzi and Secchi, 2003; Reichstein and Jensen, 2005).

Other empirical findings regarding the lognormal and Pareto distributions (Stanley et al., 1995; Axtell, 2001; Ganugi et al., 2003, 2005) have renewed interest in this research field. Using an exhaustive Business Master File for 1997, Axtell (2001) observed that FSDs are well-approximated by the Pareto distribution with exponent near unity – the so-called Zipf distribution – throughout the range of firm sizes. Kaizoji et al. (2006) used the Bloomberg database of multinational firms from the years 1995 and 2003 to analyze FSDs in terms of sales and total assets of Japanese and US companies. Those authors found that the FSD of US firms is approximately lognormal and in agreement with Gibrat's model. In contrast, the FSD of Japanese firms is clearly not lognormal and the upper tail follows the Pareto law, according to the Simon model. More recently, Cirillo and Hüsler (2009) found a Pareto distribution for 40 percent of the largest Italian firms, while Zhang et al. (2009) found that Zipf's Law accomplishes for the 500 largest Chinese firms.

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[1] From the urban economics, Gabaix (1999) and Blank and Salomon (2000) offer a solution to the puzzle of city size distribution and show that proportionate growth processes can generate Zipf's Law at the upper tail.

[2] Researching in the same area, Ganugi et al. (2003) have examined differences in normality test parameter caused by sample size.

[3] In particular, Simon (1955) and Ijiri and Simon (1977) extended Gibrat's model by introducing an entry process in which the number of firms changes and rises over time. Ijiri and Simon (1977) demonstrated that the largest firms are close to the Pareto distribution in the upper tail of the FSD.

Table 1
Summary of recent studies.

Author	Size variable	Country	Sample size	Distribution
Gallegati and Palestrini (2010)	Employees	Italy	225,000 firms	These authors argue that FSD approaches the lognormal distribution due to a "sample selection mechanism" where surviving firms obtain a higher growth rate. They also show a Pareto distribution at aggregate level, but not at sectoral levels.
Zhang et al. (2009)	Revenues	China	500 largest firms	Zipf's Law accomplishes for the 500 largest firms.
Cirillo and Hüsler (2009)	Net worth	Italy	On average 17,500 firms every year	Pareto distribution in the right tail with an exponent around 1.8.
Cefis et al. (2009)	Employees	Netherlands	More than 50,000 manufacturing firms	Lognormal distribution fits well the firm size distribution, whereas Pareto distribution fits better the upper tail. The FSD of mergers and acquisitions departs from lognormality.
Growiek et al. (2008)		Simulation	Not specified	Pareto distribution in the right tail.
Ishikawa (2008)	High-income, high-sales and positive-profits	Japan	Not specified. But it includes firms with an annual income superior to 40 million yen. Firms with negative profits are excluded	A power-law appears in the large scale region and a log-normal distribution in the middle scale region.
Crosato and Ganugi (2007)	Total assets and number of employees	Italy	5445 firms	The absence of lognormality for the total assets implies that Gibrat's Law does not hold either for the Manufacturing as a whole, or for the individual sectors. Pareto distribution in the right tail.
Gupta et al. (2007)	Sales	USA and Brazil	7518 firms in USA	Lognormal distribution does not fit well the 100 largest firms in USA or the 300 largest firms in Brazil.
Marsili (2005)	Employees	Netherlands	Around 61,000 manufacturing firms	Pareto distribution fits well the upper tail. Pareto distribution does not fit well at sectoral level.
Kaizoji et al. (2006)	Sales and assets	USA and Japan	Not specified	Lognormal distribution fits US firms, while FSD of Japanese firms is not log-normal and the upper tail follows a Pareto distribution.
Ganugi et al. (2005)	Sales and assets	Italy	11,276 firms from the mechanical sector	These authors' spatial analysis shows that lognormality is rejected for the whole country, whereas the number of acceptances of lognormality increases in the South.
Reichstein and Jensen (2005)	Sales, assets and employees.	Denmark	1017, 2737 and 3476 firms measured by sales, assets and employees respectively	Regardless of the variable, the log size distributions at sectoral level do not systematically refuse the normality test. The authors apply different normality tests.
Fujiwara et al. (2004)	Sales, assets and employees.	France and UK	Large firms: 8313 firms for assets in France, 15,776 firms for sales in France, and 15,055 firms for employees in UK	The power-law of the firm size distribution follows a Pareto distribution.
Ganugi et al. (2003)	Sales and assets	Italy	7887 firms from the ICT sector and 9822 firms from the mechanical sector	ICT sector fits well a Pareto distribution for 30% of the largest firms. For the mechanical sectors the lognormal distribution cannot be rejected. The authors apply Kolmogorov–Smirnov test.
Bottazzi and Secchi (2003)	Sales	US	More than 3000 firms	The aggregated firm size distribution shows a stable lognormal shape.
Axtell (2001)	Employees and sales	USA	More than 5 million firms	Zipf's Law adjusts to the firm size distribution.
Hart and Oulton (1997)	Employees	UK	50,441 covering all sectors	No theoretical distribution fits an observed distribution exactly.
Stanley et al. (1995)	Employees	USA	4071 manufacturing firms	Pareto distribution in the right tail.

Source: own elaboration.

From another empirical approach, Solomon and Levy (1996) showed that a power-law can also be obtained by adding a reflection condition to Gibrat's model. In other words, those authors assume that firm size is bounded from below to a threshold proportional to the average firm size. Along the same lines, Reed (2003) and Mitzenmacher (2004) described a double Pareto FSD. Reed (2003) provides a distribution that is closer to lognormal for large samples and closer to the Pareto distribution in both tails for large and small values. He calls this a double Pareto distribution.

Recently, Gallegati and Palestrini (2010) observe that it is not possible to obtain an asymptotic Pareto FSD because a cohort of surviving firms may have a positive average rate of growth. Furthermore, they develop a model where at aggregate level the FSD may have a Pareto distribution, whereas at sectoral level they obtain non-Pareto distributions.

There is, therefore, no clear response to whether the FSD follows Zipf's Law or a Pareto distribution.[4] In fact, some authors have claimed that firm size distributions are not universal (Kaizoji et al., 2006). A shortcoming of previous estimations is that they do not measure to what extent sample size affects the estimation of power-law. Gupta et al. (2007) observe that firm size distribution of Japanese and American firms must be truncated; that is, the FSD fits well with a lognormal distribution except for when larger firms are included. Furthermore, Ganugi et al. (2003) applied the Kolmogorov–Smirnov test to determine whether the FSD followed a Pareto distribution. They find that the hypothesis of the Pareto distribution cannot be rejected for 30 percent of the largest firms. Consequently, both studies shed light on the different ways that sample size can affect the FSD.

2.2. Empirical specification

Empirical literature has analyzed the relationship between firm rank and firm size in terms of market structure and firm growth (Steindl, 1965; Ijiri and Simon, 1977; Jovanovic, 1982; Ericson and Pakes, 1995; Sutton, 1997, 2007; Amaral et al., 1997). To do so, those authors have analyzed the power-law distribution. In general, a non-negative random variable X describes a power-law distribution if the *complementary cumulative distribution function* (ccdf), or $\Pr[X \geq x]$, satisfies

$$\Pr[X \geq x] \approx cx^{-\alpha}$$

where constants $c > 0$ and $\alpha > 0$.[5] It is easy to observe that the Pareto distribution is a power-law that satisfies,

$$\Pr[X \geq x] = \left(\frac{x}{k}\right)^{-\alpha}$$

for some $\alpha > 0$ and $k > 0$. If α falls in the range $0 < \alpha < 2$, then X has infinite variance. If $\alpha \leq 1$, then X also has an infinite mean. When $\alpha = 1$ this distribution is known as Zipf's Law (Zipf, 1949) or the rank size rule. The Zipf distribution is a special case of the Pareto distribution and presents the usual behavior of power-law distributions (Richiardi, 2004). Zipf's Law states that the firm size fits a power-law with an exponent approximately equal to one: the firm size is inversely proportional to the rank of the firm size. In other words, the firm size $S_{(r)}$ of a firm in the decreasingly ordered sequence of N firms with their population $S_{(1)} \geq \cdots \geq S_{(r)} \geq \cdots \geq S_{(N)}$ is inversely proportional to the rank of the size of the firm (r). Thus, a firm of rank r in the descending order of firms has a size S equal to $1/r$ times the size of the largest firms in the market.

In order to analyze the FSD, we observe the adjustment of the full sample to the lognormal distribution. Thus, we can express the Zipf distribution as,

$$r = N(1 - P(S)) = N\left(\frac{S}{k}\right)^{-\alpha}$$

where N is the number of observations above the truncation point, r is the rank and $P(S)$ is the cumulative density function of the firm size (S) and k is the truncation point. Zipf distribution is usually estimated by ordinary least squares and the regression adopts the following equation,

$$\ln r = K - \alpha \ln S + \varepsilon \tag{1}$$

where K and α are the coefficients to be estimated, and where K is a constant and ε is a random error. There are three possible results depending on the value of α. First, if α is closer to 1, the FSD is a Zipf distribution. Second, if α is larger than unity, the relationship between firm size and rank is superlinear. In other words, firm sizes diminish more than the quotient between the largest firm size and the rank that a firm occupies in the distribution. Third, if α is smaller than unity, the relationship between firm size and rank is sublinear. In other words, firm sizes diminish less than the quotient between the largest firm size and the rank.[6]

[4] For a summary of models generating different FSDs and the effect of firm dynamics on FSDs, see de Wit (2005).

[5] For details, see Mitzenmacher (2004) or Newman (2005).

[6] In the field of urban systems, Eeckhout (2004) demonstrates that if a variable adopts a lognormal distribution, the value of the parameter α from the Pareto distribution increases when the *truncation* size increases ($d\alpha/ds > 0$) and decreases when the sample size of population increases ($d\alpha/dN < 0$). Similar results are obtained by González-Val (2006) for cities and metropolitan areas in the USA during the 20th century.

Table 2
Summary statistics of Spanish manufacturing firms. Employees and sales. 2001 and 2006.

	Employees						Sales (thousands of euros)					
	2001			2006			2001			2006		
	Total	>20 years	>50 years	Total	>20 years	>50 years	Total	>20 years	>50 years	Total	>20 years	>50 years
Min.	3	3	3	3	3	3	0.003	2.8	2.8	0.46	1.12	3.7
1st quart.	6	13	18	5	10	16	317.9	1064.3	1980.8	341.3	791.28	1792.6
Median	11	27	50	10	20	43	745.0	2791.1	6333.4	774.9	2034.71	5763.6
3rd quart.	23	57	131	21	42	107	2095.8	7820.6	20,075	2120.6	5390.52	19896.4
Max.	14,775	14,775	14,775	11,955	11,955	11,955	5,963,424	5,963,424	5,963,424	5,721,771	5,122,185	5,122,185
Mean	26.5	61.9	164.5	24.2	44.3	150.3	4028.4	10923.8	38955.2	4239.5	8521.88	39566.2
Sd	116.3	234.2	811.8	99.4	166.3	690.3	42806.1	78271.5	277489.5	44665.9	61307.2	240912.2
Sk.	63.77	41.83	14.29	59.41	46.24	13.2	93.52	58.2	18.6	84.3	55.08	16.9
Kurt.	6432.3	2325.3	230.8	5632.5	2890.8	200.6	11367.3	4222.2	385.8	9191.4	4040.8	334.6
Firms	54,490	8064	547	61,455	13,260	619	54,382	8056	547	61,322	13,246	618

Sources: own elaboration from SABI database.
Min. is the minimum value; 1st quart. is the value in the 1st quartile; Median is the value at the median of the distribution; 3rd quart. is the value in the 3rd quartile; Max. is the maximum value; Mean is the mean value; Sd indicates the standard deviation, Sk. indicates the skewness index; Kurt. indicates the kurtosis index; Firms is the number of observations.

A disadvantage of estimating previous equations with OLS is that the coefficients and the standard errors are strongly biased downward in small samples (Gabaix and Ioannides, 2004). To deal with this problem, Gabaix and Ibragimov (2006, 2011) show that a shift of 0.5 for the rank is optimal and reduces the bias.[7] Therefore, our Eq. (1) becomes:

$$\ln(r-0.5) = K - \alpha \ln S + \varepsilon \tag{2}$$

Notice that the truncation occurs when a sample has been subject to a screening procedure in which all items with values lower or superior to a certain limit have been removed. Consequently, the truncation point is the limit value where the total population is screened and determines the sample size. Given that a power-law cannot define the entire size distribution, we will try to determine the sensitivity of the estimated α using the sample size, the truncation point of the distribution and the superlinear and sublinear relationship of the firm rank and its size.[8] Here, we will regard firm size as the truncation point in Section 4.2 and firm age as the truncation point in Section 5 in order to study the changes in the estimated parameter's sensitivity to both variables.

3. Data description

The dataset for this study contains more than 50,000 Spanish manufacturing firms from 2001 to 2006. The database contains information at firm level about their balance sheets in the Spanish Mercantile Register and our sample consists of those firms with more than two employees. We used the variables of the number of employees and sales to analyze the power-law of the FSD. Although both variables may define firm size, they reveal two different and important characteristics: employees indicate internal firm characteristics, whereas sales indicate the corporate performance. Table 2 reports some descriptive statistics according to variable.

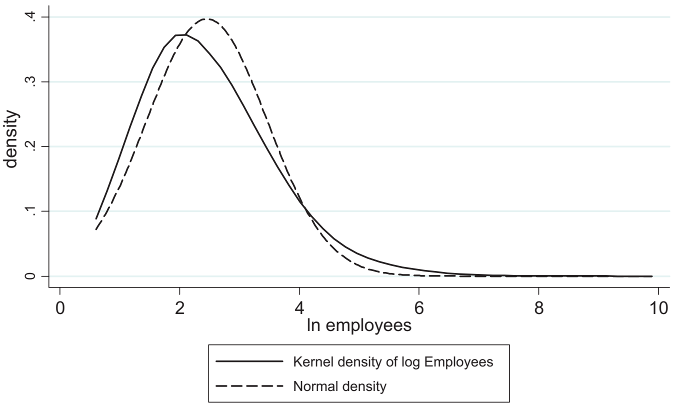
Inspection of the quartiles shows a slow shifting to the right of the distribution constant in time regardless of the variable that is considered. Also, in line with existing empirical evidence, young firms tend to be smaller than their older counterparts, regardless of the definition of size measure. Finally, mean and standard deviation increase for samples with older firms. Furthermore, both the skewness and kurtosis decrease for cohorts of older firms. Finally, these results are obtained for both variables.

Figs. 1 and 2 report the estimated FSD of the log employees and sales in 2006. We have added the normal density distribution, so that the deviation can be compared. We find significant differences among both variables. On the one hand, in line with previous empirical evidence, the FSD of log employees differs from the normal density. On the other hand, the shape of log sales is much more similar to normal density, although it is slightly biased towards the right. Nevertheless, both figures show that the density in the largest value is higher than the normal density expected in Zipf's Law. In other words, the upper tail concentrates a higher density than in the normal density. Consequently, a group of large firms is performing better than expected.

The above graphs indicate considerable widespread heterogeneity across firms, which in turn produces a skewed FSD. In fact, corporate performance seems to be closer to normal density. The graphs also confirm an upward bias of samples

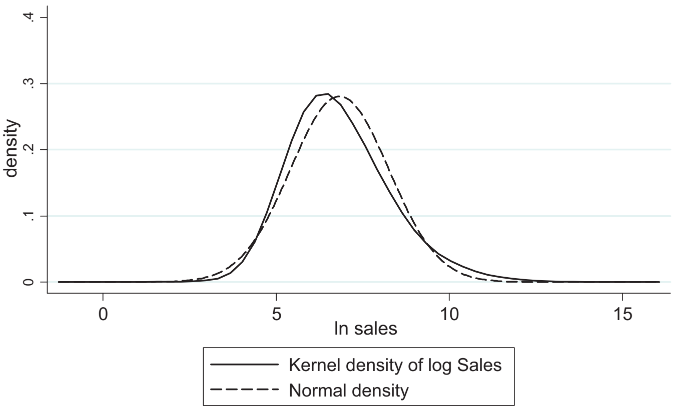
[7] Zhang et al. (2009) applied this methodology for the 500 largest Chinese firms (however, those authors express the equation as ($\ln S = K - \alpha \ln(r - 0.5) + \varepsilon$) and Peng (2010) applies this methodology to control for this bias when estimating the rank order of Chinese cities.

[8] In fact, Eeckhout (2004) points out that the parameter α is highly sensitive to the number of cities in the sample. Specifically, Zipf's Law of cities arose from the inclusion of the 135 largest US Statistical Metropolitan Areas.



Note: The curves are obtained using a normal kernel density smoother with a bandwith of 0.5

Fig. 1. Firm size distribution of ln(employees) (2006).

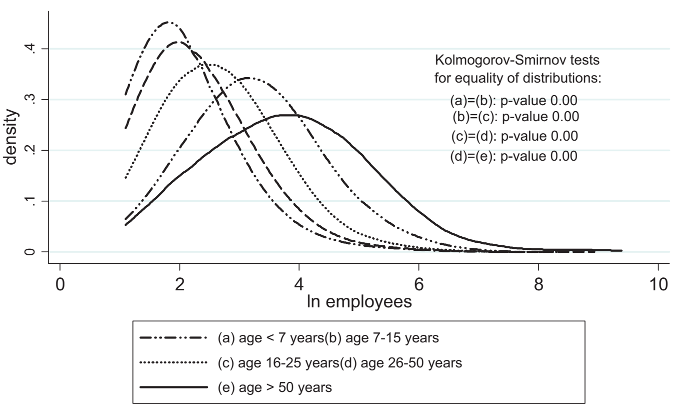


Note: The curves are obtained using a normal dernel density smoother with a bandwith of 0.5

Fig. 2. Firm size distribution of ln(sales) (2006).

among the smallest and the largest firms (kernel densities concentrate more probability in the extreme density than in the normal density).

Following Angelini and Generale (2008), we observe the evolution of the FSD according to firm age (Figs. 3 and 4). We should focus on two characteristics. First, FSD evolves towards the right when the oldest firms are considered. Second, firm age affects the evolution of the FSD similarly to both variables. These results show that firm dynamics have a rich statistical structure. Indeed, Angelini and Generale (2008) point out that there is probably a correlation mechanism between FSD and



Note: The curves are obtained using a normal kernel density smoother with a bandwdth of 0.5

Fig. 3. Firm size distribution (employees) by firm age (2006).

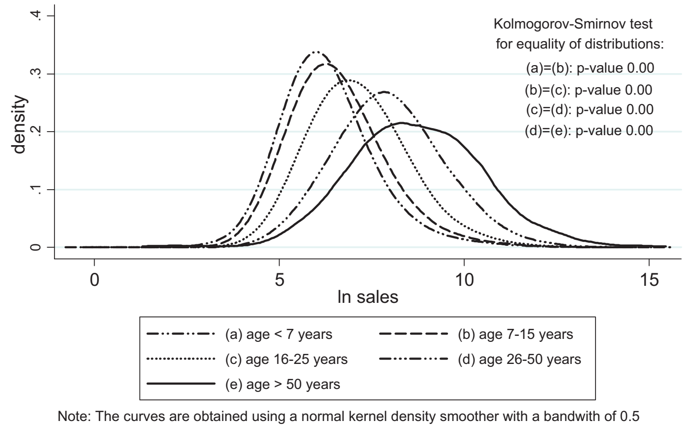


Fig. 4. Firm size distribution (sales) by firm age (2006).

Table 3
Skewness and Kurtosis tests for normality, 2001 and 2006.

	2001			2006		
	Total	>20 years	>50 years	Total	>20 years	>50 years
Log (employees)						
Prob (Skewness)	0.000	0.000	0.003	0.000	0.000	0.000
Prob (Kurtosis)	0.000	0.000	0.058	0.000	0.000	0.031
Log (sales)						
Prob (Skewness)	0.000	0.000	0.570	0.000	0.000	0.842
Prob (Kurtosis)	0.000	0.000	0.001	0.000	0.000	0.000

Sources: own elaboration from SABI database.

firm age. This mechanism stabilizes the FSD over time. Also Gallegati and Palestrini (2010) point out the existence of a process where surviving firms obtain higher average rates of growth. Finally, the figure also reports the Kolmogorov–Smirnov test results, which show that the null hypothesis of equality for the FSDs is strongly rejected for any two contiguous age classes.

Thus, previous figures confirm the stylized facts documented by Cabral and Mata (2003), Angelini and Generale (2008) and Gallegati and Palestrini (2010). In other words, the FSD is highly skewed and tends to diminish with firm age. Nevertheless, our results report differences in the FSD depending on whether we take into account the variable employees or sales.

With respect to the statistical analysis of the lognormality, Table 3 shows skewness and kurtosis tests in 2001 and 2006. In general, the null hypothesis of the symmetry of the FSDs (skewness test) is strongly rejected for the variable employees regardless of the age classes. Also we find that the logarithmic firm size is highly affected by kurtosis. However, the null hypothesis at 1 percent is not rejected for firms with more than 50 years.

If we look at the variable sales, the null hypotheses of kurtosis are significantly rejected, regardless of the firm age; the null hypotheses of skewness are also significantly rejected, except for firms older than 50 years. Thus, in general terms the lognormality of the distribution is not accepted.

If we look at the plot of log(rank) against log(size) measured in terms of employees and sales, we find that the empirical distribution does not fit well with a power-law. Figs. 5 and 6 show two interesting features. On the one hand, empirical distribution is concave with respect to the origin. On the other hand, in line with Gupta et al. (2007), there is a point of truncation with the theoretical size when Zipf's Law is applied.

We conclude our empirical investigation by estimating the Gini index. The Gini index measures the degree of unequal distribution of the firm size. Here, we show the evolution of the Gini index for employees and sales (Fig. 7). As we can see, there are two different characteristics. First, the firm size measured in terms of sales is more unequally distributed than the firm size measured in terms of employees. Second, the FSD of young firms is highly unequal and decreases over time. Although there are slight differences for employees and sales among young firms, the Gini index of both variables converges once the firm age is over 80 years (this is probably due to the scarcity of firms and the fact that those surviving firms become much more similar over time).

One question that may arise is whether there is a heterogeneous pattern among firm sizes. In order to deal with this issue, we analyze the evolution of the Gini index of employees and sales according to firm age (Figs. 8 and 9).[9] The Gini index is estimated using firms that were of a particular age in 2006. The Gini index is grouped for two different sizes: small firms

[9] We apply the code file developed by Aliaga and Montoya (1999) available for Stata.

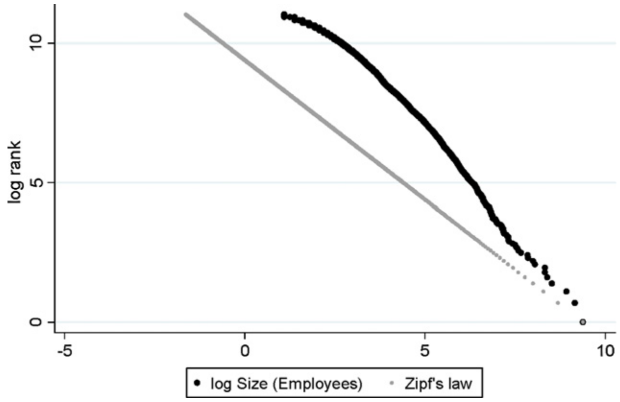


Fig. 5. Log rank vs. log size measured by employees (2006).

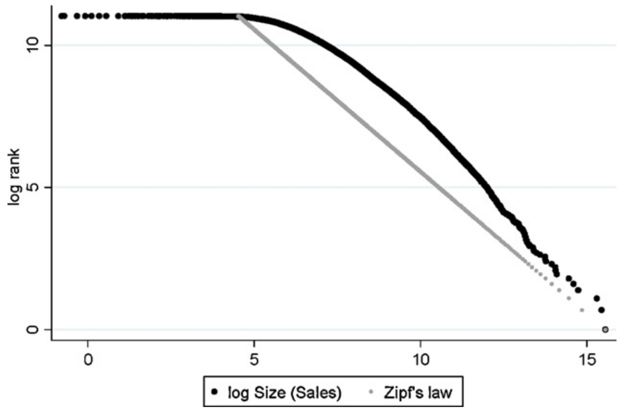


Fig. 6. Log rank vs. log size measured by sales (2006).

(those firms with fewer than 50 employees or with less than 7.6 millions of euros in sales) and large firms (firms with more than 250 employees or with more than 59.1 millions of euros in sales).

Fig. 8 shows several important differences. First, the firm size measured in terms of employees shows that the Gini index for young small firms is higher than for young large firms. This indicates that young small firms are more unequally distributed compared to large firms. Second, the Gini index for large firms becomes more volatile once they are 40 years old. Third, the Gini index for employees is more stable for small firms than for large firms. Fig. 9 shows the evolution of the Gini

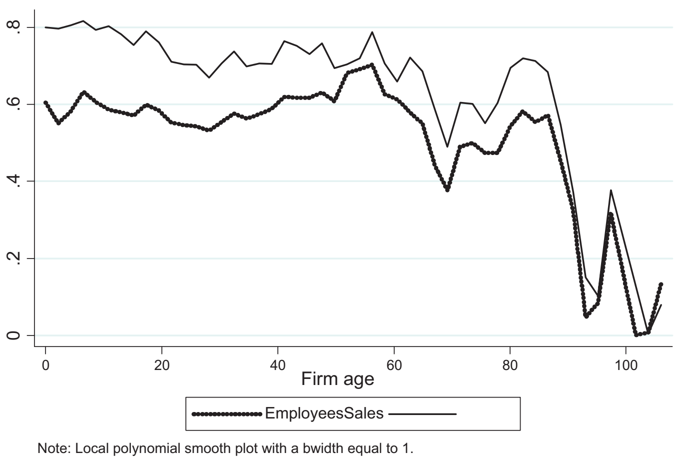


Fig. 7. Gini index for the whole database.

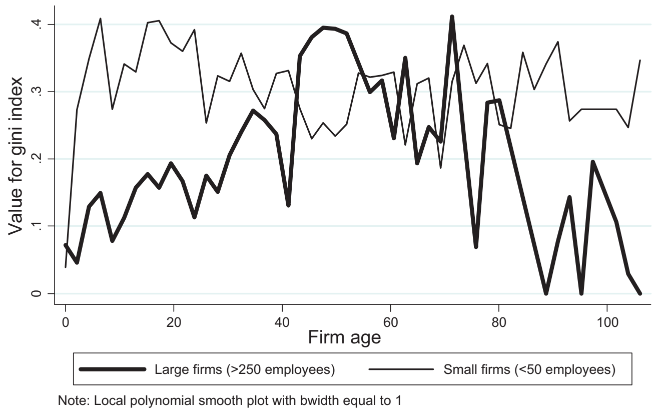


Fig. 8. Gini index for employees by firm age (2006).

index for the variable sales. Here, differences in the Gini index are not so evident in terms of firm age and in terms of firm size. Furthermore, the Gini index for small firms is higher than the Gini index for large firms.

To conclude this section, these results shed light on the different distribution of the firm size according to employees and sales and how the distribution becomes more similar when firms become more stable over time. The empirical evidence from our firms points out that a selection mechanism emerges over time which causes the FSDs to be less skewed and their size to be more equally distributed. However, the evolution of the Gini index sheds light on differences between young and old firms, small and large firms and in terms of firm's employees and sales.

4. Rolling sample results for Spanish manufacturing firms

4.1. Rolling regression methodology

According to our revision of the empirical literature, results reported in other studies indicate that there might be interactions between sample size and the power-law of a particular firm size distribution. In order to get a fuller understanding of the influence of sample size on the power-law coefficients (α), we use the rolling regression methodology to estimate the coefficients.

Although rolling regression has mostly been used to analyze the effect of the interaction between both macroeconomic variables and financial variables (Meese and Rogoff, 1988; Rousseau and Wachtel, 2002; Ibrahim and Aziz, 2003), it has also recently been applied in the context of size distribution. In particular, Peng (2010) uses it to investigate the effect of sample size and the coefficient of the Pareto distribution on Chinese cities. Hence, the main advantage of rolling regression is that it assesses the stability of the parameters because it shows the evolution of the estimated coefficients.

The technique works by ordering all observations in terms of a variable of interest (the firm size measured in terms of either employees or sales). It then estimates regressions by adding one observation at a time. In formal terms, rolling regression is a recursive least squares procedure that estimates the parameters over an increasing sequence of samples 1, 2, . . ., $N-1$, N. This gives the recursive estimates $\hat{\alpha}^{(i)}$ for $i = 1, 2, \ldots, N-1, N$. Rolling regression therefore starts with the

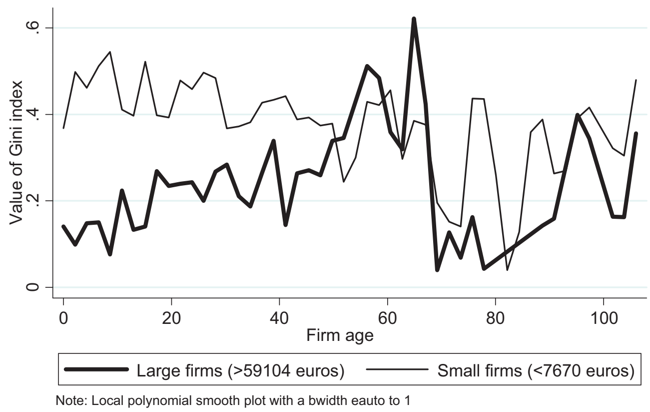


Fig. 9. Gini index for sales by firm age (2006).

Table 4
Regression results on Zipf's Law based on robust rolling regressions.

Truncation point	Employees				Sales			
	2001		2006		2001		2006	
	$\hat{\alpha}$	R^2	$\hat{\alpha}$	R^2	$\hat{\alpha}$	R^2	$\hat{\alpha}$	R^2
100	1.6520 (0.2336)*	0.9797	1.6587 (0.2346)*	0.9834	1.4189 (0.2007)*	0.9835	1.3623 (0.1918)*	0.9844
500	1.7570 (0.1111)*	0.9935	1.7679 (0.1118)*	0.9950	1.3868 (0.0877)*	0.9960	1.3954 (0.0882)*	0.9954
1000	1.6968 (0.0759)*	0.9954	1.7481 (0.0782)*	0.9973	1.3519 (0.0605)*	0.9973	1.3496 (0.0604)*	0.9966
5000	1.4463 (0.0289)*	0.9924	1.4740 (0.0295)*	0.9905	1.1745 (0.0235)*	0.9926	1.1605 (0.0232)*	0.9921
10,000	1.3822 (0.0195)*	0.9942	1.4129 (0.0200)*	0.9936	1.0797 (0.0153)*	0.9905	1.0628 (0.0150)*	0.9899
20,000	1.2906 (0.0129)*	0.9923	1.3328 (0.0133)*	0.9931	0.9735 (0.0097)*	0.9864	0.9816 (0.0098)*	0.9893
40,000	1.1007 (0.0078)*	0.9755	1.1645 (0.0082)*	0.9810	0.8194 (0.0058)*	0.9710	0.8582 (0.0061)*	0.9801
Total	0.9453 (0.0057)*	0.9473	0.9701 (0.0055)*	0.9501	0.6610 (0.0040)*	0.9128	0.6752 (0.0039)*	0.9207
N	54,490		61,455		54,382		61,322	
Truncation point when $\hat{\alpha} = 1$	49,970		58,074		17,123		17,463	

Notes: Numbers in () are corrected standard errors computed as in Gabaix and Ibragimov (2011).
* Significant at 1%.

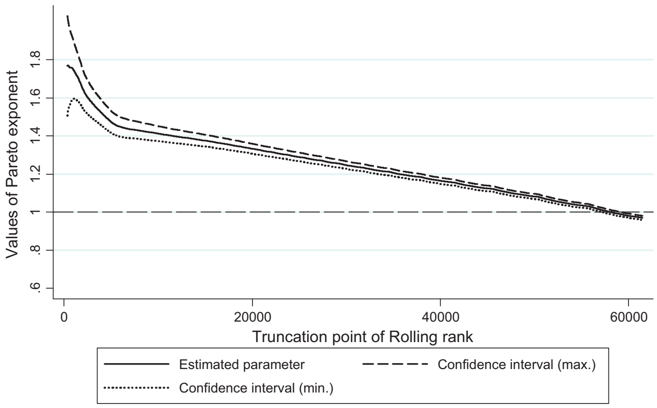


Fig. 10. Estimated parameter of employees. Spanish manufacturing firms (2006).

largest firms and includes the entire sample by the time it reaches in the last regression. It is thus a suitable methodology for assessing the stability of coefficients (Pollock, 2003).

4.2. The empirical results

In this section, we assess the parameter α, which defines the power-law of the FSDs in 2001 and 2006 in terms of the number of employees and sales [Eq. (2)]. In order to consider the effect of sample size (N) on the estimated parameter, we apply a rolling sample methodology. We then use these estimated effects to estimate the relationship between the parameter and the sample size.

The results in Table 4 show a negative relationship between the estimated coefficient and the sample size ($d\alpha/dN < 0$). This means that small samples of large firms yield higher coefficients ($\alpha > 1$) than large samples that also include smaller firms, regardless of the variable (employees or sales). In 2006, the estimated parameter of the largest 100 firms is equal to 1.6587 for employees and 1.3623 for sales, respectively. But when considering the whole sample the parameter decreases to 0.9701 and 0.6752 for both variables.

Furthermore, small samples show a superlinear relationship, although at some point in the estimation the coefficient is equal to unity. However, when the sample increases, the parameter decreases. In line with Stanley et al. (1995), the second largest firm is over half the size of the first.[10]

Figs. 10 and 11 show the evolution of the estimated parameter α for both variables in Spanish manufacturing firms. The figures highlight the decreasing pattern of the parameter when small firms are included in the sample size. However, the truncation point when $\hat{\alpha} = 1$ differs between employees and sales. For instance, in 2006 the parameter reaches unity in the

[10] A plausible explanation is that the largest firms are more homogeneous, while small firms are more heterogeneous. In the context of city size, Gabaix (1999) explained this result as the consequence of economies of scale. First, the largest cities enjoy similar economies of scale, degree of diversity and productivity, so their city size does not differ very much. Second, the inclusion of small cities decreases coefficient α. In our case, a crucial variable that may affect firm size is age.

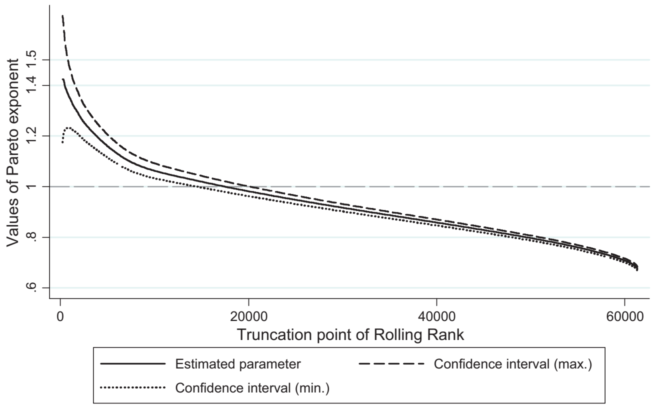


Fig. 11. Estimated parameter of sales. Spanish manufacturing firms (2006).

Table 5
The relationship between the log of the estimated Pareto exponent and the log of the sample size (2001).

	$\hat{\gamma}$	$\hat{\delta}$	R^2
2001			
Employees	1.4503	−0.1264	0.8241
	(0.0025)*	(0.0002)*	
Sales	1.4614	−0.1555	0.8589
	(0.0027)*	(0.0003)*	
2006			
Employees	1.4662	−0.1247	0.8312
	(0.0023)*	(0.0002)*	
Sales	1.3646	−0.1443	0.8777
	(0.0022)*	(0.0002)*	

* Significant at 1%.
Standard deviation is in brackets.

rank position equal to 58,074 for the employees, whereas this is reached in the rank position equal to 17,463 for sales. In other words, in 2006 the variable employees obtains a value of $\hat{\alpha}$ equal to 1 when firms have at least 3 employees and when sales are no less than 1.8 millions of euros.

All the above results indicate the differences between the pattern of the employees and sales parameters, given that the truncation point of both variables ($\hat{\alpha} = 1$) is reached at different points of the sample size. Additionally, the superlinearity suggests that the firms exceed the size predicted by Zipf's Law, while the sublinearity suggests that the firms are smaller than the size predicted by Zipf's Law.[11] In fact, our results are in line with Dinlersoz and McDonald (2009) who remark that the "important differences in the evolution of employment with respect to output distributions imply that the choice of the measure of firm size matters in investigations of firm size dynamics".

In order to check the relationship between the sample size and the parameter of rank–size Law, we ran a regression between the estimated exponent ($\hat{\alpha}$) and the sample size (SS).[12] For this analysis the following equation is estimated:

$$\log(\hat{\alpha}_i) = \gamma - \delta \log(SS_i) \tag{3}$$

Table 5 shows that using the rolling sample method to include a larger number of observations negatively affects the estimated exponents ($\hat{\alpha}$). The results confirm our expectations. The FSD is greatly affected by sample size. Consequently, the validity of Zipf's Law depends on the sample size used in a given study; increasing sample size has a negative impact on the parameter of the power-law. For instance, in 2006 the sample size has a negative impact on the estimated parameter equal to −0.1247 for the employees, whereas the firm sales parameter is more sensitive to sample size (being equal to −0.1443).

Following Peng (2010), Figs. 12 and 13 show the distributions of the coefficient $\hat{\alpha}$ obtained with the rolling regressions. We construct the efficient Epanechnikov kernel function using the optimal width for the coefficient. We observe that the kernel density function for the coefficient of employees (Fig. 12) reaches maximum density on the value equal to 1.4, but that it concentrates a large density between the interval of values equal to 1.2 and 1.4. In contrast, the kernel density function

[11] In terms of employees, in 2006 the three largest firms had 11,955 workers, 9548 workers and 7568 workers respectively. In terms of sales, in 2006 the three largest firms made 5722 million euros, 5122 million euros, and 4458 million euros respectively.
[12] This estimation does not intend to calculate the causality between both variables, but to clarify the relationship between the sample size and the value of the estimated coefficients.

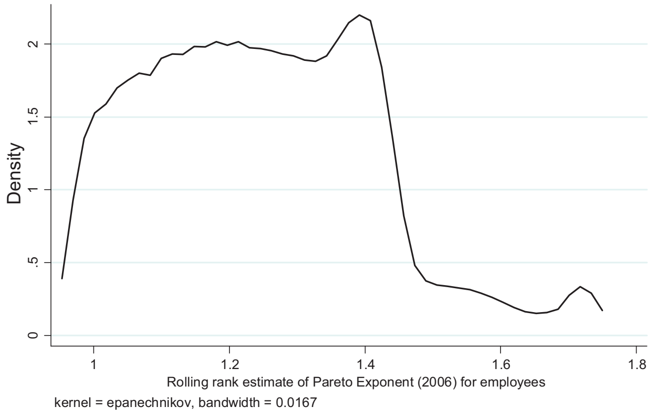


Fig. 12. Kernel density function for the Pareto exponent for employees (2006).

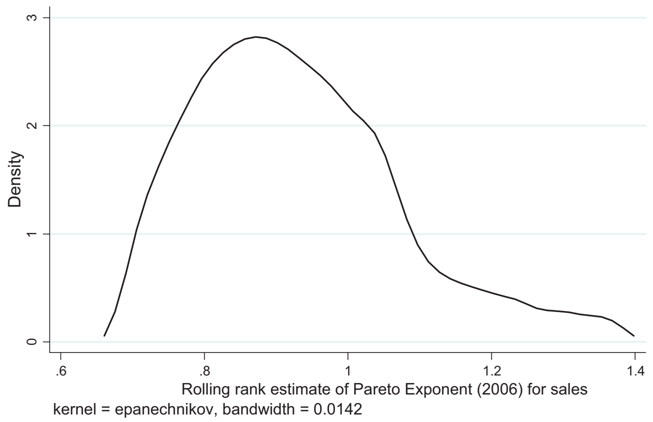


Fig. 13. Kernel density function for the Pareto exponent for sales (2006).

for the coefficient of sales appears as a unimodal density at approximately 0.9. These results confirm that the distribution of the employees and sales variables are rather different.

5. The effect of firm age on the rolling sample

After determining how sample size affects the estimated parameter of the power-law that defines the FSD of Spanish manufacturing firms, we may wonder what will be the effect of estimating the FSD in terms of different firm age. Gallegati and Palestrini (2010) remarked recently that if FSD approaches lognormal distribution over time it is because of a "sample selection mechanism". In other words, they observe that the average growth rates of surviving firms are positive. This is an alternative explanation to that of Cabral and Mata (2003),[13] in which the FSD approaches a lognormal distribution due to the existence of "financial constraints". As we have pointed out previously, the FSD differs depending on the firm age (see Figs. 3 and 4). This methodology is first to order firms from the oldest to the youngest ($Age_{(1)} \geq Age_{(2)} \geq \cdots \geq Age_{(N)}$) and to estimate small samples with the oldest firms, and then to include the youngest firms. Hence, we shift attention away from firm size and investigate the possible effect of firm age cohorts on the power-law.

Table 6 shows estimates of the parameter α for the number of employees and sales. The estimates come from estimating Eq. (2) but instead of including the smaller firms in each regression, the estimations now introduce firms according to firm age. As we expected, there is a positive relationship between the parameter α and firm age. On the one hand, in 2006 the estimated parameter for older firms (those over 50 years old) is equal to 1.2515 and 0.8934 for employees and sales, respectively. On the other hand, for those firms with more than 2 years the parameter decreases until reaching a value equal to 0.9782 and 0.6856 for both variables. This confirms the fact that young firms are smaller than Zipf's Law would suggest.

The key finding is that when considering firm age, the estimated parameter differs between the number of employees and sales regardless of the year. On the one hand, the employees variable has a superlinear relationship that changes into

[13] However, their data only covers 33,678 firms during 1991.

Table 6
Regression results on Zipf's Law using number of employees and sales based on robust rolling regressions (2001 and 2006).

Truncation point	Employees				Sales			
	2001		2006		2001		2006	
	$\hat{\alpha}$	R^2	$\hat{\alpha}$	R^2	$\hat{\alpha}$	R^2	$\hat{\alpha}$	R^2
50 years	1.2218 (0.0739)*	0.9734	1.2515 (0.0711)*	0.9728	0.9030 (0.0546)*	0.9372	0.8934 (0.0508)*	0.9375
30 years	1.1790 (0.0310)*	0.9731	1.1898 (0.0267)*	0.9725	0.8801 (0.0231)*	0.9548	0.8585 (0.0193)*	0.9529
20 years	1.1224 (0.0177)*	0.9687	1.0960 (0.0135)*	0.9656	0.8302 (0.0131)*	0.9546	0.7907 (0.0177)*	0.9531
10 years	1.0238 (0.0973)*	0.9577	1.0096 (0.0075)*	0.9555	0.7471 (0.0067)*	0.9409	0.7153 (0.0088)*	0.9356
5 years	0.9730 (0.0069)*	0.9513	0.9888 (0.0062)*	0.9524	0.6972 (0.0050)*	0.9277	0.6957 (0.0052)*	0.9292
2 years	0.9552 (0.0061)*	0.9492	0.9782 (0.0058)*	0.9512	0.6791 (0.0044)*	0.9225	0.6856 (0.0008)*	0.9254

Notes: Numbers in () are corrected standard errors computed as in Gabaix and Ibragimov (2011).
* Significant at 1%.

a sublinear relationship. On the other hand, firm sales continuously show a sublinear relationship regardless of firm age. Thus, it seems that firm age affects the number of employees and, to a lesser degree, the distribution of sales.

The empirical analysis above supports the hypothesis that firm age is a mechanism that affects the evolution of the power-law. On the one hand, our non-parametric evidence (Figs. 3 and 4) shows significant different FSDs. On the other hand, the parametric approach in Table 6 shows a high sensitiveness of firms' sales. In other words, when ordering firms from the oldest to the youngest, we obtain a different regularity than when we order firms from the largest to the smallest. From an economic viewpoint, the question remains whether firm age affects the rank rule defined by the firm size. We therefore propose Eq. (4):

$$\ln(r-0.5) = K - \alpha \ln S + \beta \ln Age \tag{4}$$

where $\ln Age$ is the firm age. In so doing, we incorporate firm age into Eq. (2) in order to control for the effect of firm experience in the market.

The results in Table A.1 show the sensitivity of firm size with respect to the rank position of the firm.[14] As can be seen the estimated parameters behave similarly. Hence, the rank–firm size relationship has been found to be very robust. We think that there is a large correlation between firm size and rank position that prevents firm age from significantly affecting this relationship (although, we have not mentioned the fact that the R^2 is quite high, which indicates a rather high correlation among both variables). Despite this disappointing result, we should not forget the importance of firm age on firm size distribution.

6. Conclusions

Using balance sheets from Spanish manufacturing firms in 2001 and 2006, this paper aims to analyze the power-law that defines FSDs. The main contributions are the following. First, from an extensive database we observed that sample size affected the estimated power-law of the FSDs. Second, we estimate the power-law for two different variables: employees and sales. Third, we analyze the effect of firm age on the estimated power-law coefficient.

Descriptive analyses show that FSD are highly right-skewed, implicating that a few large firms coexist with a large number of small firms. Furthermore, the FSD approaches the lognormal distribution when older cohorts of firms are included. This is confirmed by the kernel density of both variables: their upper tail has a greater density than in the normal FSD. Although several authors have recently analyzed the power-law that defines FSD, none of this research has looked into the effects of sample size and firm age.

The rolling regression methodology of our econometric analysis clearly shows that the estimated parameter α is non-constant. This implies that there is no constant 'power-law' between firm size and firm rank, regardless of the variable. In fact, there is a negative relationship between α and the number of observations in the estimation. This relationship changes from being superlinear (for small samples) to sublinear (for the whole sample). Therefore, sample size matters, but the parameter differs between both variables: the estimated parameter is larger for the employees variable than for the sales variable.

We conclude with several statements. First, the diversity of results obtained in the literature may reflect differences in sample size. We believe that this conclusion is in line with much of the theoretical and empirical literature available on the

[14] There is a difference between the number of firms in Tables A.1 and 4, which is due to the fact that firms with an age equal to 0 disappear when taking logs.

Table A.1
Regression results on Zipf's Law based on robust rolling regressions but controlling with firm age.

Truncation point	Employees						Sales					
	2001			2006			2001			2006		
	$\hat{\alpha}$	$\hat{\beta}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	R^2
100	1.6518	0.0068	0.9798	1.6599	0.0136	0.9840	1.4159	−0.0003	0.9845	1.3640	0.0102	0.9763
	(0.2336)*	(0.0010)		(0.2347)*	(0.0019)		(0.2002)*	(0.0001)		(0.1929)*	(0.0014)	
500	1.7540	0.0044	0.9935	1.7658	0.0041	0.9952	1.3839	−0.00002	0.9961	1.3955	0.0062	0.9880
	(0.1109)*	(0.0003)		(0.1117)*	(0.0003)		(0.0875)*	(0.0000)		(0.0883)*	(0.0004)	
1000	1.6936	0.0035	0.9954	1.7466	0.0019	0.9974	1.3496	0.0008	0.9974	1.3494	0.0024	0.9884
	(0.0757)*	(0.0002)		(0.0781)*	(0.0001)		(0.0604)*	(0.0000)		(0.0603)*	(0.0001)	
5000	1.4447	0.0049	0.9924	1.4727	0.0057	0.9906	1.1730	0.0039	0.9927	1.1602	0.0027	0.9849
	(0.0289)*	(0.0001)		(0.0294)*	(0.0001)		(0.0235)*	(0.0001)		(0.0232)*	(0.0001)	
10,000	1.3818	0.0054	0.9943	1.4125	0.0054	0.9936	1.0793	0.0080	0.9907	1.0630	0.0062	0.9828
	(0.0195)*	(0.0001)		(0.0200)*	(0.0001)		(0.0153)*	(0.0001)		(0.0150)*	(0.0001)	
20,000	1.2907	0.0088	0.9924	1.3334	0.0087	0.9931	0.9745	0.0130	0.9865	0.9826	0.0105	0.9765
	(0.0129)*	(0.0001)		(0.0133)	(0.0001)		(0.0097)*	(0.0001)		(0.0098)*	(0.0001)	
40,000	1.1027	0.0169	0.9755	1.1664	0.0144	0.9811	0.8215	0.0187	0.9710	0.8602	0.0162	0.9586
	(0.0078)*	(0.0001)		(0.0082)*	(0.0001)		(0.0058)*	(0.0001)		(0.0061)*	(0.0001)	
Total	0.9532	0.0183	0.9480	0.9756	0.0159	0.9505	0.6718	0.0174	0.9170	0.6810	0.0112	0.9228
	(0.0058)*	(0.0001)		(0.0056)*	(0.0001)		(0.0041)*	(0.0001)		(0.0039)*	(0.0001)	
N		53,568			60,778			53,469			60,653	
Truncation point when $\alpha = 1$		49,681			58,005			17,170			17,568	

Notes: Numbers in () are corrected standard errors computed as in Gabaix and Ibragimov (2011).
* Significant at 1%.

topic, which points out that including small firms in the FSD tends to be inversely related to the parameter of the power-law. Thus, there is no unique power-law defining the FSD. Second, the different patterns of the largest and the smallest firms may be the result of the firms' characteristics (economies of scale, firm age and productivity levels): the largest firms are more homogeneous than the smallest ones. Finally, firm age affects the power-law parameter when ordering firms from oldest to youngest. Although the estimated power-law parameter differs when we modify our criteria of ordering firms (in particular for the variable sales), we have shown that firm age does not directly affect the relationship between rank–size position and firm size.

This article may help to understand the relationship between sample size and firm size distribution. In a world where the firms are increasingly interconnected and the costs of clustering links are decreasing, there are many crucial factors that can help us to understand firm growth patterns. Without a doubt firm size and age are two of the internal characteristics that affect firm growth patterns.

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Appendix A.

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