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Attribution models and the Cooperative Game Theory

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Abstract

The current paper studies the attribution model used by Google Analytics. Precisely, we use the Cooperative Game Theory to propose a fair distribution of the revenues among the considered channels, in order to facilitate the cooperation and to guarantee stability. We define a transferable utility convex cooperative game from the observed frequencies and we use the Shapley value to allocate the revenues among the different channels. Furthermore, we evaluate the impact of an advertising campaign on both, the whole system and each channel.

Keywords: attribution model; Shapley value; on-line sales; Cooperative Game Theory

1. Introduction

The classic Consumer Theory (Neumann and Morgenstern, 1944) analyzes the consumer product purchasing decisions. Specifically, it studies the properties of the individuals' preferences that are transferred into a utility function, which measures the satisfaction or benefit obtained by the consumer from a specific purchasing (i.e., a combination of goods' basket). Consequently, the purchase process is obtained trough an optimization problem, where the consumer maximizes his utility function taking into account his budget constraint. Formally,

$$\begin{aligned} Max \qquad & u(x_1, x_2, ..., x_n) \\ s.a. \qquad & \sum_{i=1}^{n} p_i \cdot x_i = m \end{aligned}$$

The solution of this problem leads us to a **demand function** with a negative relationship between the quantity, x_i, and its price, p_i. It is noteworthy that the demand function plays a key-role in the literature, since its proper estimation allows us to know (i) the individuals' reactions when prices change, and, (ii) how a good reacts to its economic context. To illustrate the aforementioned comments, we present a synthetic linear demand function,

$$x_i = A_i(E) - \beta \cdot p_i$$

whose parameters have the following interpretation:

x_i: purchased quantity of good i.

$A_i(E)$: relationship of x_i with the context. This magnitude explains the interaction between the analyzed good among a large list of factors, i.e., complementary products, substitutive goods and income of the buyer.

β: individuals' reaction to changes in the price. The higher the β, the more sensitive the consumer is to changes in prices.

Quantitative research of demand functions has provided different developments on how individuals take purchasing decisions in more complex contexts. For instance, the sophisticated Steven Berry, James Levinsohn and Ariel Pakes' study about demand in automobile sector (Berry et al., 1995). Nonetheless, demand models are extremely complicated to estimate: they require a large amount of data, significant computational power and a precise econometric estimation that guarantees a proper statistical behavior. Though quantitative estimation of demand function is such a difficult task, its econometric specification sheds additional light regarding the purchasing process. Therefore, from an empirical perspective, a demand function takes the following expression,

$$x_i = A_i(E) - \beta \cdot p_i + \varepsilon_i,$$

where the error ε_i is introduced to our simple linear demand. The error plays a fundamental role on the consumers' purchasing mechanism, as it provides a random component in the original model. On the one hand, the new term measures the consumers' response to news and different stimulations related to a non-deterministic way of the purchasing dynamics of x_i. On the other hand, the qualitative impact of ε_i is extraordinary, since it explains how exogenous phenomena might alter the purchasing decision. Indeed, the better modelling of the error term has improved the understanding of some economics fields. For instance, Engle (1982) dramatically enhances the comprehension of Financial Markets thanks to his ARCH model, which is a refinement on how to model ε_i in stock returns time series (see Bollerslev, 1987).[1]

In our context, ε_i has two main implications. Firstly, it transforms the initial model to a more realistic approach, since it relaxes the strong rationality hypothesis of the consumer theory. Secondly, ε_i tell us that applying the right amount of positive pressure, individuals may be exogenously influenced in order to increase the sales of a good (see Scott, 1976, Tybout, 1978 and Prabhu and Stewart, 2001, among others). Therefore, even if the demand function remains unknown a marketer can raise his success via publicity, i.e., advertisements.

In this regard, it is noteworthy that in the digital media era, consumers are viewing ads nearly everywhere, trough several different marketing channels (organic search, email, display ads, social media, for instance). With a high volume of conversions, a marketer may wonder what channel is more efficient and what channels must be reinforced to improve future sales. Here is when the concept of *Atribution* arises naturally. Attribution concept was originated in psychology ans was introduced in Marketing during the early 70s. Within that period of time we find several studies which try to evaluate the success of different marketing techniques (Settle and Golden, 1974; Swinyard and Ray, 1977; Mizerski, 1978; Li and Kannan, 2014; and Kannan et al., 2016, among others), but the concept of marketing attribution has evolved with the departure from traditional selling strategies. Nowadays, attribution may be defined as the quantification of the influence that each advertising impression has on a consumers' conversions.

The current approach, trough the use of Game Theory, provides some

[1] Such sophistication was awarded with the Economics Nobel Prize in 2003.

insights into the problem of measuring the attribution, which is not a trivial task. Several attribution methods are available today, where the more commonly used are those based on single and fractional source. Nonetheless, the problem concerning to these methods is that according to the chosen model, a bias that generates a conflict between the different digital marketing channels may not be avoid. Henceforth, more complex perspectives are available to overcome this issue. At this point, it is noteworthy that digital marketing channels are not isolated, indeed there exists positive feedback between them increasing the likelihood of purchasing. Consequently, **Google Analytics 360** has based its new Data-Driven Attribution model on *Cooperative Game Theory* and the *Shapley Value*.

In this context, the current paper tries to improve the weakness of the allocation methods that are based on sequences of touch points (last click, first click, time decay, among others). Concretely, we use the Cooperative Game Theory to propose a fair distribution of the revenues among the considered channels, in order to facilitate the cooperation and to guarantee its stability. In doing so, firstly, and due to the features of the analysed problem, we define a convex game, taking into the account the observed frequencies. Secondly, we use the Shapley value to allocate all the revenues among the different channels. Finally, we study the actual impact of an advertising campaign on both the whole system and each channel.

The remained paper is organized as follows. Section 2 and 3 introduce the Cooperative Game Theory and the Shapley value, respectively. 4 provides the definition of the sale channels game, analyses its properties, and implements it to a specific case (with and without advertising campaigns). Finally, Section 5 concludes.

2. Cooperative games

There exists a large number of social and economic situations where the agents are strategic dependent, i.e., each agent's outcome is influenced by the other agents' decision. The Economics field that studies such situations is Game Theory, whose influence in Economic modelling is extraordinary. Specifically, the current paper focus on the situations where cooperation among agents is necessary and mutually beneficial, such as the formation of a cartel among companies, or the financial support trough the *crowdfunding*. In doing so, we use the Cooperative Game theory, which not only modelizes the cooperation among agents (in terms of gains and costs), but also, it provides solutions to determine the way to share the benefits obtained from the cooperation among agents. These situations, where collaboration and conflict of interest arise naturally, are called *games*, and the agents, *players*, who may be individuals, nations, political parties, associations, companies, etc.

It is noteworthy that the importance of the role to be developed by Game Theory is to provide, from a quantitative rather than a qualitative point of view, the objective tools that promote the cooperation and solve potential conflict. Specifically, the analysis of such situations from a formal and an axiomatic point of view becomes essential as the complexity of the situation and the assets to be distributed increases. Accordingly, next we introduce the key concepts that are necessary to understand the game proposed and developed in the current paper.

2.1. The characteristic function

As aforementioned, we study how to allocate the gains obtained trough a sale on Internet among the different channels involved in the sale process. Given the nature of the object to be analyzed, we assume transferable utility, i.e., players negotiate with a perfectly divisible good (money). In a transferable utility cooperative game we have a finite set of players $N = \{1, 2, \ldots, n\}$, which can be grouped into 2^N subsets of N, called bf coalitions. Coalitions are represented in capital letters $S, R, T, ...$ and the corresponding lower case letter will represent the number of players in the coalition; so, the coalition S has s players. In addition, the coalition without players are called the empty coalition and it is represented by $\varnothing$. The game assigns to each coalition a numeric value $v(S)$. Formally,

Definition 1. *A transferable utility cooperative game is a pair (N, v), where N is a finite set of players and v is a function, the characteristic function*

$$v : 2^N \to \mathbb{R}$$

that assigns to each coalition a numeric value, $v(S) \in \mathbb{R}$ for all $S \subseteq 2^N$, that we call the worth of the coalition. Assuming that $v(\varnothing) = 0$ (the worth of the empty coalition is zero). Hereinafter, let G^N denote the class of all games with player set N.

From now on, for the sake of comprehension, **we will refer to cooperative games with transferable utility simply as games**.

Many of the results proposed by the Game Theory are based on the fulfillment of certain properties by the characteristic function. In order to reinforce the suitability of the game we propose in Section 4, we present some properties that are considered as minimal requirements:

Monotony: If the number of players in the coalition increases, the benefits should not decrease.

Superadditivity: The union of coalitions with no common players is beneficial.

Convexity: The higher the coalition, the higher each player's marginal contribution.

3. The Shapley value

Once the grand coalition N is achieved, in order to cooperate and maximize each agent's gains, how will the profits be distributed among the players? Solving this question, many solutions concepts are proposed in the literature (see Matsumoto and Szidarovszky, 2016, for instance) satisfying two minimal requirements:

- Individual rationality: An allocation x satisfies individual rationality if each player receives a payoff greater or equal to what can be guaranteed on his own, without cooperating with any one else, i.e., $x_i \geqslant v(i))$ for all $i \in N$.
- Efficiency: An allocation $x(N)$ is efficient if it distributes the worth of the grand coalition $v(N)$ among all players, i.e., $x(N) = x_1 + \cdots + x_n = v(N)$.

Among all the proposed solutions, we introduce the Shapley value, due to the fact that it considers the concept of marginality (a key issue in our framework), and it satisfies a set of properties that may be considered as compulsory conditions in our context. In doing so, it is important to analyze what marginality is.

The marginal cost, $c(N) - c(N \backslash \{i\})$ of a certain player $i \in N$ is an important indicator when allocating the total costs of a project among its participants. In situations where profits are distributed, the marginal distribution of a player $i \in N$ to a given group is $v(N) - v(N \backslash \{i\})$, indicating the player's contribution i to the common benefit.

Usually, assigning the marginal contribution to each player is not efficient, and, therefore, it is not a solution. In order to avoid the problem of inefficiency, we may consider that players join the coalition following a certain ordering, and, then, consider each players' marginal contribution. Note that this distribution, called the vector of marginal contributions, is efficient, but depends on an arbitrary ordering of the players. This distribution is called the marginal contribution vector associated with θ and we denote it by $m^\theta(v)$.

Definition 2. *Let* $\mathbf{m}^\theta(v) \in \mathcal{R}^n$ *be the* **vector of marginal contributions** *associated to an ordering* $\theta = (i_1, \dots, i_n)$*, where, for each player* $i \in N$*,*

$$m^\theta_{i_1}(v) := v(i_1),$$

$$m^\theta_{i_2}(v) := v(i_1, i_2) - v(i_1),$$

$$\dots$$

$$m^\theta_{i_n}(v) := v(i_1, \dots i_n) - v(i_1, \dots i_{n-1}).$$

In order to solve the problem of the arbitrariness of the ordering of the players, Shapley (1953) proposes a distribution that considers all possible orderings. Specifically, he assumes that each ordering has the same probability of being considered, therefore, he considers the average of all the marginal contributions according to all possible orderings. Formally,

Definition 3. *let* (N, v) *be a game, the* **Shapley value** *of this game,* $\phi(v) = (\phi_1(v), \dots \phi_n(v))$*, for each player* $i \in N$*, is defined as,*

$$\phi_i(v) := \frac{1}{n!} \sum_{i \in S_n} m^\theta_i(v)$$

where the summation is applied on the set S_n *of all the orderings, and* $m^\theta(v)$ *is the associated vector of marginal contributions.*

Note that the Shapley value selects an efficient allocation, always exists and it is unique. That is, whatever the characteristics of the game is, we can always compute it, and the result is a univalued distribution. Furthermore, if the game is supperaditive, then the Shapley value is individual rational; and, if the game is convex, the Shapley value belongs to the set of solutions whose allocation can not be improved by any group of players, known as the Core of the game (Gillies, 1953; Shapley, 1953). Consequently, we can argue that the allocation proposed by the Shapley value is somewhat stable, since no player or group of players could improve on it. Therefore, it is noteworthy that the Shapley value considers the following two key features:

Marginal contribution: the Shapley value does not share the individual revenue according to the worth of the individual coalitions, it measures the individual contribution of each player to each coalition. Therefore, the players are awarded by their contribution in each of the possible cases.

Temporal sequence: the ordering in which the player joint to the coalition is a conflict issue. For avoiding it, the Shapley value computes each player's marginal contribution taking into the account all the possible orderings.

Finally, Shapley (1953) shows that this solution is the unique solution that jointly satisfies the following properties:

- Efficiency: The Shapley value distributes all gains or costs among players.
- Symmetry: If two players make equal contributions to the game, i.e., if they are substitutes, they should receive the same amount.
- Dummy player: If a player does not provide any additional benefit to the other players, he should not receive any additional payment. In terms of the game if the player's marginal contribution is equal to zero, then he must receive an allocation equal to his individual worth.
- Additivity: The player's allocation for a sum of two games is the sum of the player's allocations for each individual game.

4. Cooperative Games applied to the rationing problem among channels

This section implements the aforementioned theoretical framework to the case where the gains obtained trough a sale on Internet should be distributed among different channels. In doing so, firstly, we provide the formal definition of the associated game; secondly, we apply this model for the three-channels case; and, finally, we extend the model to measure the impact of advertising campaigns.

4.1. Model definition and properties

In the current application, each possible channel is a player of N, and for each sub-set of channels $S \subseteq N$, a coalition, we know the number of sales, denoted by $I(S)$. By using these data, we define an associated cooperative game where the worth of each coalition is the number of sales that all the coalition's channels can achieve. Formally,

Definition 4. *Given a set of players N the* **sale channels game** *is a TU-game $(N, v) \in G^N$, such that, for each coalition $S \subseteq N$,*

$$v(S) = \sum_{R \subseteq S} I(R), \ \ I(R) \geqslant 0.$$

Given this definition, we show that the sale channels game is convex, which is an essential property to defend the implementation of the Shapley value, as mentioned in Section 3.

Proposition 1. *For all $(N, v) \in G^N$ a sale channels game, it is a convex game.*

Proof. We must prove the convexity condition: for each coalition $S \subseteq T \subseteq N \backslash \{i\}$, $v(S \cup \{i\}) - v(S) \leqslant v(T \cup \{i\}) - v(T)$.

On the one hand, we know that:

$$v(S \cup \{i\}) - v(S) = \sum_{R \subseteq S \cup \{i\}} I(R) - \sum_{R \subseteq S} I(R) = \sum_{R \subseteq S \cup \{i\}, i \in R} I(R).$$

On the other hand,

$$v(T \cup \{i\}) - v(T) = \sum_{R \subseteq T \cup \{i\}} I(R) - \sum_{R \subseteq T} I(R) = \sum_{R \subseteq T \cup \{i\}, i \in R} I(R).$$

Given that $S \subseteq T$, for each $R \subseteq S \cup \{i\} \subseteq T \cup \{i\}$ with $i \in R$ and $I(R) \geqslant 0$, implies that $\sum_{R \subseteq S \cup \{i\}, i \in R} I(R) \leqslant \sum_{R \subseteq T \cup \{i\}, i \in R} I(R)$ fulfilling the convexity condition. ■

Additionally, since every convex game is superadditive, the defined game does.

4.2. A 3-channels case

Suppose that for a period of time we study the sale success for three channels: Direct, Organic and CPC (hereinafter, players 1, 2 and 3, respectively). To apply the proposed model, we need not only the independent sales of each channel, but also the sales obtained by the interaction of the channels (see Table 1).

Channels	$I(R)$
1	19786
2	20837
3	24008
12	898
13	822
23	822
123	194

Table 1: **Data base.** Data obtained from each of the channels and their interaction.

Given this information about the frequencies, the associated cooperative game is built by Definition 4. In order to simplify the implementation of this game and its computation, we apply the matrix format through,

$$B \times \varphi = v(S)$$

where B is a binary squared matrix of dimension $2^n - 1$, containing the coefficients related to $I(R)$ and taking into the account if the players are part of the coalition S; φ is a vector composed by the values $I(R)$; and, $v(S)$ denotes the worth of the coalitions. Applying it for a 3-players game, we obtain the following expression,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} I(1) \\ I(2) \\ I(3) \\ I(12) \\ I(13) \\ I(23) \\ I(123) \end{bmatrix} = \begin{bmatrix} v(1) \\ v(2) \\ v(3) \\ v(12) \\ v(13) \\ v(23) \\ v(123) \end{bmatrix}$$

Hence, it is easy to show how the coalitions are built. For instance,

$$v(2) = I(2), v(12) = I(1) + I(2) + I(12), v(23) = I(2) + I(3) + I(23),$$

$$v(123) = I(1) + I(2) + I(3) + I(12) + I(13) + I(23) + I(123),$$

that is,

$$v(2) = 20837, v(12) = 19786 + 20837 + 898, v(23) = 20837 + 24008 + 822,$$

$$v(123) = 19786 + 20837 + 24008 + 898 + 822 + 822 + 194.$$

By using the total number of frequencies obtained in the Table 1, we obtain the value of each coalition (see Figure 1, and Table 2).

Coalition	$v(S)$
1	19786
2	20837
3	24008
12	41521
13	44616
23	45667
123	67367

Table 2: **Worth of the *sales channels game* characteristic function.** Data from Table 1.

At this point, we study how to distribute the worth of the grand coalition $v(N)$ among the three channels composing the game. As aforementioned (Section 3), the Cooperative Game Theory proposes alternative fair ways of sharing $v(N)$ that induce the cooperation among all the players.

Hence, organizing the information in this way, we can check that the Shapley value (ϕ_i) is the average value of each player's marginal contributions, taking into the account all the possible orderings. Specifically, in our example, we obtain the Table 4. Note that the Shapley value proposes an allocation that ensures to each player a larger amount than the worth of the individual coalition (individual rationality), and the sum of the all the payments corresponds with the worth of the grand coalition (efficiency).

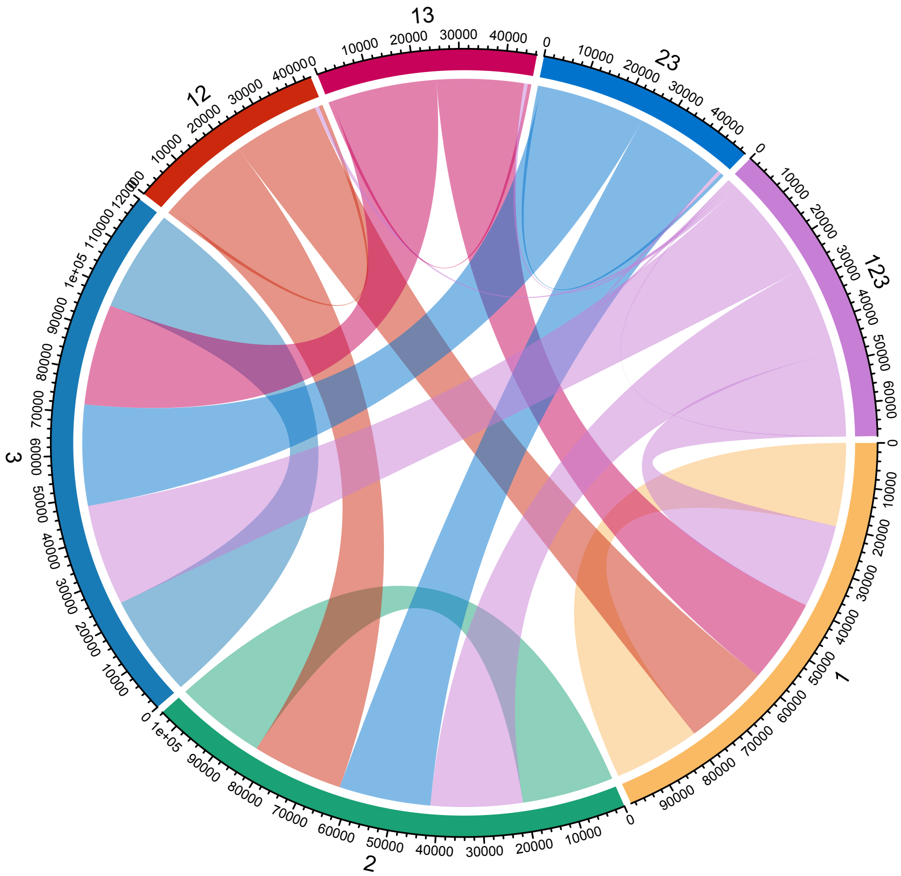

Figure 1: **Graphic representation of a three-players sales channels game.** The areas which correspond to the individual coalitions show each player's influence on the total game. The intermediate coalitions areas represent the position and the final composition, approximately.

4.3. The effect of advertising campaigns

It is noteworthy to see that the introduced approach is perfectly useful to analyse if an advertising campaign is effective, since we can find out the

Arrival ordering, θ	$m_1^\theta(v)$	$m_2^\theta(v)$	$m_3^\theta(v)$
$\theta=(1,2,3)$	$v(1)$	$v(12)-v(1)$	$v(123)-v(12)$
$\theta=(1,3,2)$	$v(1)$	$v(123)-v(13)$	$v(13)-v(1)$
$\theta=(2,1,3)$	$v(12)-v(2)$	$v(2)$	$v(123)-v(12)$
$\theta=(2,3,1)$	$v(123)-v(23)$	$v(2)$	$v(23)-v(2)$
$\theta=(3,1,2)$	$v(13)-v(3)$	$v(123)-v(13)$	$v(3)$
$\theta=(3,2,1)$	$v(123)-v(23)$	$v(23)-v(3)$	$v(3)$
$\phi_i(v)$	$\frac{1}{3!}\sum m_1^\theta(v)$	$\frac{1}{3!}\sum m_2^\theta(v)$	$\frac{1}{3!}\sum m_3^\theta(v)$

Table 3: **Formal computation of the Shapley value for the three-players game.** The rows represent the players' arrival ordering(θ) and the columns their marginal contribution (m_i^θ).

Player	**Shapley value** (ϕ_i)
1	20710.67
2	21761.67
3	24894.67

Table 4: **The Shapley value applied to the data obtained from Table 2.**

actual impact of it in the whole system.[2]

To initiate our analysis, consider that we examine the frequencies of three channels (Social, Organic and CPC) data during the week before the campaign is applied. After this week, an advertising campaign starts and we obtain updated data from the different channels (see the second column of Table 5 and, graphically, Figure 2). Comparing both results, note that the campaign has been a success. The issue here is how to compute its actual effect on the whole system and of the sales of each of the three channels. In doing so, we introduce the advertising campaign as a fourth player in the proposed game (the fourth column in Table 5).

By applying Definition 4 on the introduced data, we can compute the worth of each coalition $v(S)$ by using the observed frequencies (see Table 6). Note that, for computing the worth of all the coalitions with four players, the matrix B becomes larger,

[2]Due to the brevity of the advertising campaigns, the use of methods that are only based on frequencies may make hard to distinguish the actual effects of the the advertising campaign.

Channels	**Pre-Campaign**	**Post-Campaign (3)**	**Post-Campaign (4)**
1	1105	1368	1105
2	592	666	592
3	120	1183	120
4			74
12	741	2174	741
13	753	761	753
14			263
23	313	1439	313
24			74
34			1063
123	426	1383	426
124			1433
134			8
234			1126
1234			883

Table 5: **Sales frequencies, $\mathbf{I(R)}$, Pre-Campaign and Post-Campaign.**

$$\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&1&0&0&1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&1&0&0&1&0&0&0&0&0&0&0&0 \\ 0&1&1&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&1&0&1&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&1&1&0&0&0&0&0&1&0&0&0&0&0 \\ 1&1&1&0&1&1&0&1&0&0&1&0&0&0&0 \\ 1&1&0&1&1&0&1&0&1&0&0&1&0&0&0 \\ 1&0&1&1&0&1&1&0&0&1&0&0&1&0&0 \\ 0&1&1&1&0&0&0&1&1&1&0&0&0&1&0 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix} \times \begin{bmatrix} I(1) \\ I(2) \\ I(3) \\ I(4) \\ I(12) \\ I(13) \\ I(14) \\ I(23) \\ I(24) \\ I(34) \\ I(123) \\ I(124) \\ I(134) \\ I(234) \\ I(1234) \end{bmatrix} = \begin{bmatrix} v(1) \\ v(2) \\ v(3) \\ v(4) \\ v(12) \\ v(13) \\ v(14) \\ v(23) \\ v(24) \\ v(34) \\ v(123) \\ v(124) \\ v(134) \\ v(234) \\ v(1234) \end{bmatrix}$$

Next, in Table 7 we apply the Shapley value to each situation, using the frequencies of three channels (Social, Organic and CPC).

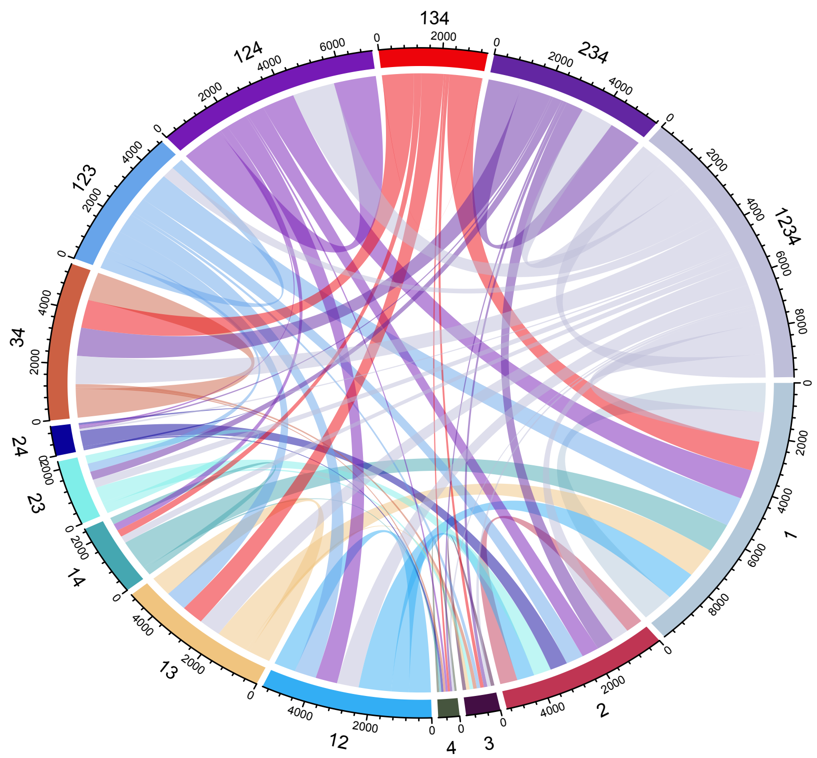

Figure 2: **Graphic representation of a four-players sales channels game.** The areas which correspond to the individual coalitions show each player's influence on the total game. The intermediate coalitions areas represent the position and the final composition, approximately. Note the complexity increment of the relationships among players.

Channels	Pre-Campaign	Post-Campaign (3)	Post-Campaign (4)
1	1105	1368	1105
2	592	666	592
3	120	1183	120
4			74
12	2438	4208	2438
13	1978	3312	1978
14			1442
23	1025	3288	1025
24			740
34			1257
123	4050	8974	4050
124			4282
134			3386
234			3362
1234			8974

Table 6: **Characteristic function, v(S), for the three proposed games.**

Channel	Pre-Campaign	Post-Campaign (3)	Post-Campaign (4)
Social	1994	3296.5	2826.58
Organic	1261	2933.5	2371.75
CPC	765	2744	1925.25
Campaign	-	-	1850.42
$v(N)$	4050	8974	8974

Table 7: **The Shapley value for the three considered scenarios.**

It is noteworthy to see that all the channels have obtained profits by the advertising campaign. Specifically, when we analyse the situation as a four-players game, we can see clearly its impact. In this sense, note that $v(4) = 74$, but the fourth player's Shapley value is larger (1850). This difference suggests that the fourth player's marginal contributions are large. Computing the differences for each channels in each situation we obtain that all the channels have increased their sales, specially the CPC channel (see the Table 8). Finally, observe that the sum of the differences among the Shapley values of the three channels coincide with the Shapley value of the advertising campaign (Table 7).

Channel	Post-Campaign (3)	Post-Campaign (4)	Dif.	Dif. %
Social	3296.5	2826.58	469.92	25.40
Organic	2933.5	2371.75	561.75	30.36
CPC	2744	1925.2500	818.75	44.25
Total	8974	7123.58	1850.42	100

Table 8: **Differences of the Shapley value between the two situations.**

5. Final remarks

The aim of the current paper is to provide to the reader a compatible framework with the Google approach. Additionally, we show its application to the attribution context, and we evaluate the impact of a digital campaign on the purchasing process. In doing so, we define a way to transfer the consumers' conversions into a convex cooperative game, and we apply the Shapley value to our data.

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