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Self-interest and Equity Concerns: A Behavioural Allocation Rule for Operational Problems

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Abstract

In many economic situations, individuals with different bargaining power must agree on how to divide a given resource. For instance, in the dictator game the proposer has all the bargaining power. In spite of it, the majority of controlled experiments show that she shares an important amount of the resource with the receiver. In the present paper I consider how behavioural and psychological internal conflicting aspects, such as self-interest and equity concerns, determine the split of the resource. The individual allocation proposals are aggregated in terms of altruism and value for the resource under dispute to obtain a single allocation. The resulting allocation rule is generalized to the n-individuals case through efficiency and consistency. Finally, I show that it satisfies a set of desirable properties. The obtained results are of practical interest for a number of situations, such as river sharing problems, sequential allocation and rationing problems.

Keywords: Behavioural operational research; Sharing rules; Altruism; Equity concerns; Self-interest.

JEL classification: C91, D03, D63, D74.

1. Introduction

In many economic situations of interest individuals must agree on how to divide a given resource. However, not all individuals have the same bargaining power and consequently some individuals are in a better position than others. Situations of this kind tend to be the rule rather than the exception (see, Ambec and Sprumont, 2002; Curiel et al., 1989; Herings and Predtetchinski, 2012; Moulin, 2000; Kilgour and Dinar, 2001). For instance, in the river sharing problem, upstream individuals benefit from a better strategic location than downstream individuals. Similarly, sequential allocation and rationing problems establish priorities among individuals. In this type of problems the equal split of the resource is unlikely to result because the best positioned individuals claim a larger share of the resource. In those contexts, the difficulty in implementing a practical solution arises when assessing each individual position and how to incorporate it into a negotiated solution that would be accepted by the involved parties (Babcock et al., 1995; Babcock and Loewenstein, 1997).

In order to resolve this situation, I start by noting that the dictator game (Kahneman et al., 1986) and the ultimatum game (Güth et al., 1982) have structures that are similar to the problem described above. The proposer (the individual with more bargaining power, higher priority or upstream) may or may not share a given resource with the responder (the individual with less bargaining power, no priority or downstream). Despite the fact that rationality predicts that individual behaviour should be mainly self-interested, the vast majority of controlled experiments show that agents do not act in accordance with this postulate (Aguiar et al., 2008; Camerer, 2003; Engel, 2011). The main message of these and other studies (discussed below) is that individuals promote altruism.

The question is whether we can use the knowledge accumulated through these games to solve actual operational problems having a similar sequential structure and in which the restrictions faced by the proposer are mostly ethical and not material.

Since the proposer can freely consume the full resource without any punishment, the starting point is to understand when and why people share the available resource in social dilemmas of this kind. This has been a key issue in all social sciences (Fehr and Fischbacher, 2003; Engel, 2011; Dreber et al., 2014). In this context, the individuals' willingness-to-give is usually interpreted as altruism (Camerer, 2003): a sacrifice of one's resources for the benefit of others.

This internal trade-off between self-interest and equity concerns has motivated a vast body of literature. Ravallion et al. (2004) note that extreme unequal agreements raise concerns about social and political stability. In this sense, the large majority of subjects avoid being considered as unfair (Brañas-Garza, 2007; Reuben and Van Winden, 2010; Rodriguez-Lara and Moreno-Garrido, 2012), regardless of their altruistic concerns (Dana et al., 2006).

Several theories have been put forward to explain these empirical regularities. They consider behavioural motives such as altruism, fairness, reciprocity, inequity and guilty aversion as possible explanations for the observed departures from pure selfish behaviour. For instance, Bolton and Ockenfels (2000) and Fehr and Schmidt (1999) - through the inequality aversion theory - defend that individuals dislike inequity, which is measured by deviations from the equal split. Hence, individuals are willing to forgo some monetary payoffs in order to help others that are worse off. Charness and Rabin (2002) suggest that people have maximin preferences. They care about their own payoff but they also want to maximize the minimum social welfare.[1]

In the present paper, I do not specify an explicit utility function. Expected utility models require assumptions about individuals' utilities with implications for the results (see Baron (2000) for a discussion on these and other related issues). This aspect distinguishes the model in the present paper from the existing models in the literature (Bolton and Ockenfels, 2000; Charness and Rabin, 2002; Fehr and Schmidt, 1999; Kőszegi and Rabin, 2006). Furthermore, there is no social planner, welfare or fairness maximization objective which are commonly assumed in the resource allocation literature (Kaplow and Shavell, 2000; Thomson, 2001, 2015). Instead, the objective is to offer a practical but consensual solution that can be applied in real life operational problems. This deliberate practical and applied focus is akin to that advocated by those working within the growing area of be-

[1] Engelmann and Strobel (2004) compare the relative performance of these theories. They conclude that efficiency and maximin preferences have greater explanatory power than inequality aversion. Edgeworth (1881), Griesinger and Livingston (1973) and Loewenstein et al. (1989) are examples of other early attempts to formalize the individuals' trade-off between their own payoffs and the payoffs of others. In the same line, Sanfey et al. (2003) find that low offers activate emotional brain areas (insula and dorsolateral prefrontal cortex) associated with judgement, planning, and conflict resolution, see also Reuben and Van Winden (2010). Other approaches, such as the guilt aversion theory, posit that people feel guilty if their behaviour falls short of the others expectations (Charness and Dufwenberg, 2006; Ellingsen et al., 2010).

havioural Operational Research (Franco and Hämäläinen, 2016; Hämäläinen et al., 2013).

It is noteworthy that during the last decades we have observed a growing number of experimental studies in bargaining and conflict resolution but without a correspondence in terms of theoretical models. There have been almost no practical or operational solutions to real life problems derived from these studies, with a few exceptions that are mostly in contexts outside resource allocation problems (Brailsford and Schmidt, 2003; Brailsford et al., 2012; Franco and Meadows, 2007; Morton and Fasolo, 2009; Rouwette et al., 2011). The present paper attempts to fill this gap - the fundamental argument is that the division of a resource should be based on evidence about human behaviour in similar circumstances.[2]

In line with these comments, I propose a simple theory in which individuals are simultaneously self-interested and equity concerned. Hence, the interception of these conflicting but non-contradictory aspects frames the heterogeneity of individual proposals between the most egalitarian and the most self-interested allocation - the most and the least altruistic allocation, respectively.

Subsequently, the individuals' proposals are aggregated according to the behavioural principle of empathy, and the political concepts of participatory democracy and representativeness. These universal principles are captured through a distribution over the set of reasonable proposals. Furthermore, since the level of altruism may depend on the importance given to the resource under dispute, I consider a distribution that aggregates all possible valuation.[3]

I start by analyzing the two-individual case. Specifically, I consider a

[2]The same reasoning can be extended to other contexts in order to establish the basis for new research that would aim at seeking practical solutions to real life problems. Similar ideas have been put forward by Bendoly et al. (2006) or Gino and Pisano (2008) in the context of operations management.

[3]The proposed theory is normative. It suggests how some resource should be allocated between individuals according to a set of desirable principles (Baron, 2004; Brams and Taylor, 1996; Thomson, 2001). These principles are grounded on empirical evidence of actual behaviour in resources allocation problems (rather than on ideal models of behaviour). In the behavioural operational research literature the paper locates within the "behaviour in models" stream (Brocklesby, 2016; Franco and Hämäläinen, 2016).

binomial/Poisson model to capture the different levels of altruism and the importance that individuals assign to different values of the resource. This case shows some interesting insights. For instance, the proposed allocation rule endogenously replicates the empirical evidence, suggesting that the value of the resource is determinant for the individual's willingness to give (List and Cherry, 2008; Engel, 2011; Sefton, 1992): the higher the value of the resource, the lower the desire to be altruistic, and vice versa.

Subsequently, I generalize the allocation rule for the n-individuals case by imposing *efficiency* and some form of *consistency* (Moulin, 2000; Thomson, 2011; Young, 1987). The result is a practical rule founded on behavioural arguments that determine how a resource should be split among individuals in a society that is characterized by some degree of altruism and expected valuation for the resource under dispute. It is also shown that the proposed sharing rule satisfies some relevant properties that are considered as basic in the resource allocation literature (Thomson, 2001, 2015).

The obtained results are of interest for several practical problems. For instance, the n-individuals sequential structure in the present paper is similar to that in the river sharing problem (Ambec and Sprumont, 2002; Kilgour and Dinar, 2001), in bargaining problems in which each individual share of the resource is sequentially determined (Curiel et al., 1989; Herings and Predtetchinski, 2012), or in rationing problems (Moulin, 2000). To the best of my knowledge, the proposed sharing rule is the first attempt to introduce behavioural and psychological considerations into this type of problems.

The exposition concludes with an illustrative example taken from the river sharing literature and discusses some relevant issues for applied work. Throughout the paper there is an intentional balance between realism and simplicity that can help researchers and practitioners in operational work.

The paper is organized as follows. Section 2 describes the problem and some behavioural aspects. Section 3 considers the two individuals case. Section 4 generalizes to the n-individuals case. Section 5 analyses some properties. Section 6 provides an example and guidelines for applied work. Section 7 concludes.

2. The Problem and behavioural Characteristics

Consider a scenario in which individual 1 (the proposer) decides on how to divide some resource with value $y \in [0, \infty)$ between herself $y_1 \in [0, y]$ and individual 2 (the responder), $y_2 = y - y_1$. Empirical evidence shows that the value of the resource is determinant in the individuals' altruistic decisions (List and Cherry, 2008; Engel, 2011; Sefton, 1992). Consequently, y is assumed to follow some distribution with different proposers giving different value to the resource under dispute.

The individual 1 is free to make any choice whatsoever. Individual 2 may irrationally disagree and block the possibility of a negotiated solution (Babcock et al., 1995; Babcock and Loewenstein, 1997) but in practical terms is unable to change the decision of individual 1. In this context, what would be the most adequate split that considers the individuals' different bargaining positions and has chances of being accepted by the involved parties? In order to answer this question, I start by considering individual behavioural aspects such as equity concerns and self-interest. Then, I discuss their aggregation into an allocation rule.

Individual behavioural Properties

As aforementioned, a large body of empirical literature suggests that (i) individuals promote altruism (Fehr and Fischbacher, 2003; Engel, 2011); and (ii) the value of the resource is determinant in the individuals' decisions (List and Cherry, 2008; Engel, 2011; Sefton, 1992). In this context, I aim at constructing a simple theory that is in line with these two robust observations.

Further, since the number of behavioural considerations that can play a role in the individual's decision is likely to be uncountable or at least difficult to determine, I introduce the two principles that are most prominent in the empirical literature (Camerer, 2003; Diekmann et al., 1997; Engel, 2011; Ottoni-Wilhelm et al., 2014): equity concerns and self-interest.

Equity concerns support the idea that individuals should be treated in the same way if they are in equal positions (Adams, 1963). However, individuals are usually differentiated into one or several dimensions. In the context of the present paper, individual 1 is better positioned than individual 2.

Definition 1. *An individual is* ***equity concerned*** *if* $y_1 = y/2 + \varepsilon \sim y_1 = y/2 - \varepsilon$, *and* $y_1 = y/2 \pm \varepsilon \succeq y_1 = y/2 \pm \varepsilon'$ *with* $\varepsilon \leqslant \varepsilon'$.

An equity concerned individual must be indifferent between two allocations that are equally distant from the equal split allocation, $y_1 = y/2$. For instance, the allocations $y_1 = 0$ and $y_1 = y$ are equity equivalent because these two are at the same distance from the midpoint. However, in addition, an equity concerned individual must have a preference for more even allocations (see Ottoni-Wilhelm et al. (2014) for a discussion on altruistic motives). This preference induces a bias towards more even allocations.

Besides equity concerns, there are other aspects that characterize an individual behaviour. One such aspect is self-interest, that is, individuals tend to keep a higher amount of the resource for themselves.

Definition 2. *An individual is **self-interested** if* $y_1 \geqslant y_2$.

Contrary to equity concerns, self-interest induces a bias towards more unequal allocations. This self-serving bias is a common source of disagreement in bargaining problems (Babcock et al., 1995; Babcock and Loewenstein, 1997).

The consideration of Definitions 1 and 2 implies that self-interest and equity concerns become conflicting but not contradictory aspects. The tension between these two aspects is always present in resources allocation problems (Diekmann et al., 1997). The result is that individuals forgo part of their own interest to promote altruism. This observation is supported by the vast majority of the theoretical and experimental literature in dictator and ultimatum games referred in the Introduction (for example Aguiar et al., 2008; Camerer, 2003; Engel, 2011; Reuben and Van Winden, 2010; Rodriguez-Lara and Moreno-Garrido, 2012; just to mention a few).

Finally, in the context of negotiation, I assume that the proposer split is observed by the responder and by third parties. Consequently, such a proposal can be the object of social censure and criticism. In this context, Reuben and Van Winden (2010) note that unfair individual actions have associated higher intensity of emotions such as shame and guilt, while, Ravallion et al. (2004) argue that extreme unequal agreements raise concerns about social and political stability. Hence, no individual would propose an allocation that is unanimously unacceptable, that is likely to cause disagreement and result in noisy negotiations.

Definition 3. *An allocation proposal is **inadmissible** if it is completely self-interested, i.e.,* $y_1 = y$.

Aggregation Properties

The intersection of equity concern, self-interest and admissibility determines the interval $[y/2, y)$. Therefore, allocations in which the proposer receives less than half of the resource are excluded, i.e., $y_1 < y_2$. These are more extreme forms of altruism that might be rational at the individual level in some contexts but are not representative of aggregated behaviour (Luhan et al., 2009).

Definition 4. *An allocation is* ***reasonable*** *if* $y_1 \in [y/2, y)$.

There is a correspondence between the individuals and their proposed allocations, i.e., each individual is characterized by an allocation $y_1 \in [y/2, y)$.

Finally, the aggregation rule receives as input every reasonable proposal.

Definition 5. *An aggregation rule is* ***universal*** *if its domain is the set of all reasonable allocations.*

The idea of universal allocation is connected with the behavioural principle of empathy, besides the political concepts of direct or participatory democracy and representativeness. The aggregation rule considers "everybody's" point of view. In other words, each allocation is considered in the decision process and receives a strictly positive weight. I wish the allocation rule in the present paper to inherit this property. This argument is common in the context of judgment aggregation (List, 2009).

3. Aggregation Rule ($n = 2$)

The objective of the two individuals' analyses is to present the baseline allocation that expresses the aggregate behaviour of a heterogeneous society involved in a dictator-type game. Later, the procedure is generalized to n-individuals.

In what follows, I describe the modelling approach.

Reasonable Allocations and Resource Representation

Consider a resource with value $y > 0$ that is divisible into smaller amounts of equal and constant size $\varepsilon > 0$. For instance, y may denote a prize denominated in euros while ε may denote one euro. Since ε is fixed, in order to capture the impact of an increase in the value of y; let $y(r) \equiv (2r+2)\varepsilon$ with $r = 0, 1, ...$, capture the effect of variations in the value of the resource under dispute. For instance, a small r implies that the dispute is not very important for the proposer; therefore, we must expect it to be more generous. On the other hand, a large r implies the opposite, the resource under dispute is very important in relative terms.

Note that the discrete resource $y = 2\varepsilon, 4\varepsilon, ...$, grows two units per unit increment on r. There is no loss of generality. In doing so, we are able to include the most equal allocation $y_1(r) = y(r)/2 = (r+1)\varepsilon$ (i.e., the lower bound on the set of reasonable allocations) and exclude the most unequal allocation $y_1(r) = y(r) = (2r+2)\varepsilon$ (i.e., the inadmissible allocation). After scaling on ε, this approach is equivalent to, for example, $y = 1\varepsilon, 2\varepsilon, ...$ (Osório (2016) apply a similar discretization argument).

In general terms, for a given r, the elements in the set of reasonable allocation proposals of Definition 4 can be described by the proposer allocation $y_1(r)$ because the responder allocation $y_2(r)$ is simply the difference $y(r) - y_1(r)$. Therefore, the elements in the discrete set of reasonable allocation are given by $\{(2r+1-j)\varepsilon\}_{j=0}^{r}$ with $r = 0, 1, ...$

In order to better understand the construction and composition of the discrete set of reasonable allocation proposals, consider the following illustrative example.

Example. *For* $r = 0$, *we have* $y(0) = 2\varepsilon$, *which guarantees that we always have an allocation that belongs to the set of reasonable allocation proposals, i.e.,* $y_1(0) = y(0)/2 = \varepsilon$. *For* $r = 1$, *we have* $y(1) = 4\varepsilon$, *which implies that we have two allocation proposals belonging to the set of reasonable allocation proposals, i.e.,* $y_1(1) = 3\varepsilon$ *and* $y_1(1) = y(1)/2 = 2\varepsilon$ *(where* $y_2(1) = 1\varepsilon$ *and* $y_2(1) = 2\varepsilon$, *respectively). For* $r = 2$, *we have* $y(2) = 6\varepsilon$, *which implies that we have three allocation proposals belonging to the set of reasonable allocation proposals, i.e.,* $y_1(2) = 5\varepsilon$, $y_1(2) = 4\varepsilon$ *and* $y_1(2) = y(2)/2 = 3\varepsilon$ *(where* $y_2(2) = 1\varepsilon$, $y_2(2) = 2\varepsilon$ *and* $y_2(2) = 3\varepsilon$, *respectively).*

The example also shows that proposals become more distributed, creating a smooth movement towards more unequal offers as the value of the resource increases. For instance, for $r = 0$, $r = 1$ and $r = 2$ the most even allocation proposal represents 1/1, 1/2 and 1/3 of the total number of reasonable allocation proposals, respectively. In other words, as the value of the resource increases, the weight given to the most even allocations decreases, contrary to the weight given to the less even allocations. The discretization endogenizes this process.

Later, the distribution over the set of reasonable allocations re-establishes continuity and allows the consideration of any value of $y > 0$. An alternative approach would be to consider a continuous space from the beginning. However, such an approach does not capture the importance that the value of the resource has on the individuals' altruistic decisions.

Since the interest is on the individual share and not on the absolute value, I can write the reasonable allocation proposals in terms of the share on the total resource $s(r) = y_1(r)/y(r) \in [1/2, 1)$. Therefore, the elements in the set of reasonable allocation shares is given by $\{(2r+1-j)/(2r+2)\}_{j=0}^{r}$ with $r = 0, 1, ...$ This representation is independent of ε because there is a cancellation effect - the numerator and denominator are simultaneously scaled by the same number.

Example (cont.). *In terms of shares on the total resource, for* $r = 2$, *we have* $s(2) = 5/6$, $s(2) = 4/6$ *and* $s(2) = 3/6$ *(where* $1 - s(2) = 1/6$, $1 - s(2) = 2/6$ *and* $1 - s(2) = 3/6$, *respectively).*

These examples also show the diversity of possible allocations; some allocations are more altruistic than others, i.e., more equity concerned and/or less self-interested.

Reasonable Allocations and Distributions of Resources

Definition 5 of universality requires that each allocation belonging to the set of reasonable allocation proposals should be considered. I capture the universal concept by assuming that the mass of allocations y_1 (or individuals) follow some probability distribution $F(.)$ with support in the set of reasonable allocation proposals $[y/2, y)$. In a discrete setting, I must consider some probability mass function $f^r(j) > 0$ over $j = 0, 1, ..., r$ (because j is the varying component of $y_1 \in \{(2r+1-j)\varepsilon\}_{j=0}^{r}$). The higher the mass $f^r(j)$ associated with a given value of j, the higher the number of individuals supporting or proposing the allocation $(2r+1-j)\varepsilon$.

Note that the higher (respectively, lower) the value of j, the higher (respectively, lower) the share of the endowment given to the responder (respectively, proposer), i.e., the more equity concerned and/or less self-interested is the proposer.

Individuals also differ in terms of the value that they give to the resource under dispute $y \in [0, \infty)$ (see Section 2). In other words, some individuals attribute a low value to the resource, while others attribute a high value. For instance, it is natural to expect that a wealthy individual will value less the resource under dispute than a poor individual. Therefore, I consider a distribution of possible resource values, i.e., y follows some distribution $G(.)$ with support in $[0, \infty)$. In a discrete setting, I must consider some probability mass function $g(r) > 0$ over the varying component $r = 0, 1, ...$ of $y = (2r + 2)\varepsilon$.

Note that, if there is no uncertainty about the value of the resource under dispute the function $g(r)$ places all mass (i.e., $g(r) = 1$) in a given value of $r = 0, 1, ...$

Since the total resource is a random variable, the following result is expressed in terms of the expected share $s \equiv E(y_1/y)$ of the total resource.

Proposition 1. *The universal share of the total resource is,*

$$\begin{aligned} s &= \int_{[0,\infty)} \int_{[y/2,y)} \frac{y_1}{y} dF(y_1|y)\, dG(y) \\ &= \sum\nolimits_{r=0}^{\infty} g(r) \left(\sum\nolimits_{j=0}^{r} \frac{2r+1-j}{2r+2} f^r(j) \right), \end{aligned} \tag{1}$$

where $r = 0, 1, ...,$ *and* $j = 0, 1, ..., r.$

The proof follows from simple statistical concepts.

The result is a unique allocation founded on behavioural arguments that depart from other classic approaches (Thomson (2001), surveys this literature).

Expression (1) has the following interpretation. In a society characterized by altruistic and valuation preferences distributed according to $F(.)$ and $G(.)$, respectively, the proposer (individual 1) should consume at most the share of the resource $s \in [1/2, 1)$, given by expression (1), and leave the remaining share for the responder (individual 2). Simultaneously, since the society internalizes the universal share as the standard, any other consumption above s is less consensual, and may be the object of social censure and criticism.

The binomial/Poisson model

The choice, of the distribution over the set of reasonable allocation (equity concerns and self-interest) and the distribution over the value of the resource, has some implications for the predictions of the model. Nonetheless, we can calibrate the resulting model to the empirical data if we have control over some distributional parameters. In what follows, for practical and operational proposes, I consider two particular distributions.

Among discrete distributions with finite support, the binomial is particularly suitable to model the distribution of reasonable allocation proposals y_1 because it aggregates equity concerns and self-interest into a single measure of altruism. Consequently, we can vary this measure according to the characteristics of the population. If $f^r(j)$ is binomial $B(r, \alpha)$, then $\alpha \in [0, 1]$ represents the aggregate *degree of altruism*. Therefore, when α is small (respectively, large) the distribution places more mass on more unequal (respectively, even) allocation proposals, reflecting a less (respectively, more) altruistic society.

Assumption 1 $f^r(j) = \binom{r}{j}(1-\alpha)^{r-j}\alpha^j$, for $j = 0, 1, ..., r$.

On the other hand, among discrete distributions with infinite support, the Poisson is particularly suitable to model the distribution of resource values y. This distribution is particularly flexible and is frequently used in practice. In our context, it enables us to characterize the population in terms of the expected value of the resource under dispute. If $g(r)$ is Poisson $Poisson(\lambda)$, then the parameter $\lambda \in [0, \infty)$ captures the aggregate *expected value of the resource*. Consequently, when λ is large (respectively, small) the resource under dispute has high (respectively, low) value in aggregate terms which will reflect a higher interest for it by the individuals in this society.

Assumption 2 $g(r) = \lambda^r e^{-\lambda}/r!$, for $r = 0, 1, ...$

In operational terms, the binomial and Poisson distributions are particularly simple and robust because other distributions are particular cases. Another practical implication is that the universal share of the total resource in Proposition 1 can be described with only two parameters, i.e., α and λ. In applied work, these two parameters can be estimated or chosen to match the real data (see Section 6).

Corollary 1. *Under Assumptions* 1 *and* 2, *the universal share of the total resource is,*

$$s = 1 - \frac{\alpha\lambda + (1-\alpha)\left(1 - e^{-\lambda}\right)}{2\lambda}, \tag{2}$$

where $\alpha \in [0,1]$ *and* $\lambda \in [0,\infty)$.

The result is a convenient analytical expression.

Properties

The individual 1 share $s \in [1/2, 1)$ of the resource has the following properties.

Proposition 2. *Under Assumptions* 1 *and* 2;

a) $\partial s/\partial\lambda > 0$ *(*s *is concave in* λ*) for all* $\alpha \in [0,1)$ *and* $\lambda \in [0,\infty)$.

b) $s \downarrow 1/2$ *if* $\lambda \downarrow 0$.

c) $s \uparrow 1 - \alpha/2$ *if* $\lambda \uparrow \infty$.

d) $\partial s/\partial\alpha < 0$ *(*s *is linear in* α*) for all* $\alpha \in [0,1]$ *and* $\lambda \in [0,\infty)$.

e) $s \uparrow 1 - \left(1 - e^{-\lambda}\right)/2\lambda$ *if* $\alpha \downarrow 0$.

f) $s \downarrow 1/2$ *if* $\alpha \uparrow 1$.

Figure 1 illustrates these properties. In what follows I briefly comment on them.

Part a) states that the average allocation is more (respectively, less) unequal if the material value of the resource is high (respectively, low). This result is empirically supported by List and Cherry (2008), Engel (2011) and Sefton (1992), among others. However, the equity concerns component of the individuals' behaviour impacts on this movement. For that reason s is concave in λ.

Part b) states that if the value of the resource under dispute is sufficiently small ($\lambda \downarrow 0$), then the society is more likely to support allocations closer to the equal split. This observation suggests that some allocation proposals that appear in the data, and which are considered to be altruistically motivated, are simply the result of uninterested behaviours. In other words, some experiments might be undermined by uninterested subjects that may look altruistic when in fact they do not care or value enough the resource under dispute. Consequently, we might be overestimating altruistic behaviour and we may justify why some individuals propose less than half of the resource to themselves - a behaviour considered irrational in most contexts.

Part c) states that if the expected value of the resource under dispute is sufficiently high ($\lambda \uparrow \infty$), then the average proposer demands more self-interested proposals. The only aspect that restricts the proposer from having the full resource is the society level or sense of altruism that is captured by parameter α.

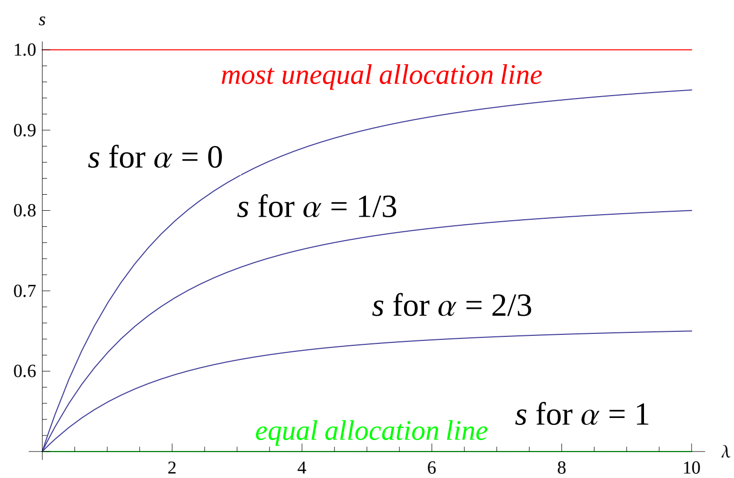

Figure 1: The universal share of the total resource s as a function of λ for different values of α. Note that s converges to the *equal allocation line* for $\alpha \uparrow 1$, but does not converge to the *most unequal allocation line* for $\alpha \downarrow 0$ unless $\lambda \uparrow \infty$.

Part d) states that altruism favors the acceptance of more even proposals. However, the result depends on the material value of the resource.

Part e) states that if the aggregate degree of altruism in the society is low ($\alpha \downarrow 0$), then more unequal allocations obtain more social support.

Part f) is also consistent. In spite of the material value of the resource under dispute, extreme levels of altruism ($\alpha \uparrow 1$) may justify an equal division.

Note that a necessary condition for the emergence of the most even allocation is that either $\lambda \downarrow 0$ or $\alpha \uparrow 1$. On the other hand, the most unequal allocation requires simultaneously that $\lambda \uparrow \infty$ and $\alpha \downarrow 0$, independently of the order of the limits.

Finally, I note that these properties are robust to distributional assumptions other than the binomial/Poisson specification.

4. Consistency and the n-individuals case

Until now we have seen the two individuals' case. The ultimate objective is to extend the allocation found in Proposition 1 to general sequential allocation problems with n-individuals. I do it by imposing a *consistency* property (Moulin, 2000; Thomson, 2011; Young, 1987).

The n-individuals sequential allocation structure is similar to an n-individuals dictator game. First, individual 1 consumes part of the resource and passes the remainder to individual 2. Then, the latter individual consumes part of the received resource and passes the remainder to individual 3, and so on. Other examples with a similar structure are river sharing problems (Ambec and Sprumont, 2002; Kilgour and Dinar, 2001), sequential bargaining problems in which each individual share is sequentially determined (Curiel et al., 1989; Herings and Predtetchinski, 2012), or rationing problems (Moulin, 2000).

The *consistency* argument is the following. If the first individual keeps a share of the resource and passes the remainder to the second individual, then if there was a third individual this should be treated in the same way by the second individual. In other words, the second individual should keep a share of what has been passed to her and leave the remainder for the third individual. However, the first individual (as well as the second individual) cannot be indifferent to the addition of a third individual. Therefore, in the $n = 3$ case, individual $i = 1$ must accept a share of the total resources lower than s, obtained in Proposition 1, and individual $i = 2$ must accept a share of the total resource lower than $1-s$. In other words, we require the resulting allocation rule to be globally consistent, as well as efficient.

The following procedure describes how I impose *consistency* in a general setting with n-individuals. Let s_i be the share of individual $i = 1, ..., n$ in the total resource. In order for the previous reasoning to be consistent, observe that when $n = 2$ the share of the individual $i = 1$ on the total resource is $s/(1 - s)$ times larger than the share of the individual $i = 2$. In other words,

$$s_1/s_2 = s/\left(1 - s\right), \tag{3}$$

with $s_1 + s_2 = 1$, by efficiency. The solution to this system is $s_1 = s$ and $s_2 = 1 - s$, which by *consistency* is equal to Proposition 1. Similarly, for $n = 3$, *consistency* requires that we must satisfy Equality (3), but also the equality:

$$s_2/s_3 = s/\left(1-s\right),$$

with $s_1 + s_2 + s_3 = 1$, by efficiency. The solution to this system of three equations and three unknowns is given by,

$$s_1 = \frac{(s/(1-s))^2}{1 + s/(1-s) + (s/(1-s))^2},$$

and

$$s_2 = \frac{s/(1-s)}{1 + s/(1-s) + (s/(1-s))^2},$$

with s_3 obtained by efficiency.

Following this procedure, the resulting solution generalizes for any number of individuals. The proof follows from the previous discussion.

Proposition 3. *The universal consistent share of individual $i = 1, ..., n$, on the total resource is,*

$$s_i = \frac{(s/(1-s))^{n-i}}{\sum_{k=1}^{n}(s/(1-s))^{k-i}} = \frac{(2s-1)\left(s/(1-s)\right)^{n-i}}{(1-s)\left((s/(1-s))^n - 1\right)}, \tag{4}$$

for $n = 1, 2, ...$, where s is given by (1).

In our n-individuals allocation problem, *consistency* imposes that pairwise allocations must be linked through a generalized equal treatment principle.

Finally, I replace the analytical expression (2) into expression (4) of Proposition 3 to obtain the n-individuals generalization of Corollary 1. The conclusions found in Proposition 2 are similar, with the necessary adaptations to the n-individuals case. Table 1 provides a numerical example.

5. Additional Properties

In this section, I analyze the properties of the n-individuals sharing rule of Proposition 3 by adapting to our context some basic properties that have

	$n = 2$	$n = 3$	$n = 4$	$n = 5$
$i = 1$	0.637	0.527	0.480	0.457
$i = 2$	0.363	0.301	0.274	0.261
$i = 3$		0.172	0.156	0.149
$i = 4$			0.089	0.085
$i = 5$				0.048

Table 1: The individual $i = 1, ..., n$ universal consistent share of the resource for $\alpha = 0.6$, $\lambda = 3$, and $n = 2, 3, 4, 5$.

been considered as general and minimal requirements of fairness in the literature (Thomson, 2001; Thomson, 2015). Throughout the analysis of these properties, I provide a deeper understanding of the proposed sharing rule.

Anonymity implies that the identity of agents should not matter. Therefore, the share of the resource received by each agent should depend only on their relative position, and not on who owns it.

Anonymity. For all $y \in [0, \infty)$, each $\pi \in \Pi^N$ and each $i = 1, 2, \ldots, n$, $s_{\pi(i)} = s_i$, where Π^N denotes the class of bijections from N into itself.

Order preservation (Aumann and Maschler, 1985) is considered as one of the minimal requirements of fairness. If an individual is in a better strategic position than another individual, the former cannot receive less than the latter, i.e., s_i decreases with i.

Order preservation. For all $y \in [0, \infty)$, and each $i = 1, 2, \ldots, n$, $s_i > s_{i+1}$.

Claims monotonicity implies that if an individual improves her position, then she should receive at least as much as she did initially. In other words, if an agent has more bargaining power, she should receive more.

Claims monotonicity. For all $y \in [0, \infty)$, each $i = 1, 2, \ldots, n$ and each $i' = 1, 2, \ldots, n$, if an individual i improves her position, i'-th $\leqslant$ i-th, then $s_{i'} \geqslant s_i$.

Resource monotonicity (Curiel et al., 1987; Young, 1987) says that if the endowment increases, each agent should receive at least as much as she did initially.

Resource monotonicity. For all $y, y' \in [0, \infty)$, if $y' \geqslant y$, then $y'_i \geqslant y_i$, for each $i = 1, 2, \ldots, n$.

Note that, although an increase in y reduces the proposer incentives to offer more equal allocations, the absolute contribution $y_i = s_i \cdot y$ is increasing with y. In order to see it, consider the following numerical example from the **binomial/Poisson** model for the case $n = 3$.

Example 1. *Suppose that* $n = 3$, $\alpha = 0.6$ *and* $\lambda = 3$. *In this case* $E(y) = 2 \times 3 + 2 = 8$. *The individuals* $i = 1, 2, 3$, *universal consistent shares of the resource are* $\{0.53, 0.30, 0.17\}$, *respectively (see Table 1), and the corresponding allocations are* $\{4.22, 2.41, 1.37\}$, *respectively. Now suppose that* $\lambda = 4$. *In this case* $E(y) = 2 \times 4 + 2 = 10$. *The individuals* $i = 1, 2, 3$, *universal consistent shares of the resource are* $\{0.55, 0.29, 0.16\}$, *respectively (see Table 1), and the corresponding allocations are* $\{5.48, 2.94, 1.58\}$, *respectively.*

The absolute value of each individual allocation always increases while the relative share s_i increases with λ for $i = 1$ but decreases for $i = 2, 3$. In connection with the discussion in the previous sections, the higher the expected value given to the resources, the less altruistic becomes individual $i = 1$. Individual $i = 2$ is also affected by this effect. However, her share decreases mostly because, simultaneously, the remainder that is passed to her also decreases.

Population monotonicity establishes that if new individuals arrive to the problem, each individual that was initially present should receive at most as much as she did initially, i.e., s_i decreases with n.

Population monotonicity. For all $y \in [0, \infty)$, and each $i = 1, 2, \ldots, n$, if $n' \geqslant n$, $s_i \geqslant s_i'$.

No advantageous merging (O'Neill, 1982) states that individuals may not be better off by forming a group (the associated n necessarily decreases), instead of being alone. In other words, if two or more individuals are presented in the problem as a single agent, they would not obtain any benefit from it.

No advantageous merging. For all $y \in [0, \infty)$, and each $i, j = 1, 2, \ldots, n$, $s_i + s_j \geqslant s_{i'}$, where $i' \leqslant i < j$, for each $i' = 1, 2, ..., n-1$.

Note that if two individuals decide to merge, the formed group will occupy the best position of both individually. Otherwise, it is obvious that if the position of an individual is worse off, the share will decrease, as stated by *claims monotonicity*.

Analogously, *no advantageous splitting* (O'Neill, 1982) states that individuals become worse off by dividing their position, instead of acting as a single agent.

No advantageous splitting. For all $y \in [0, \infty)$, and each $i = 1, 2, \ldots, n$, $s_i \geqslant s_{i'} + s_{j'}$, where $j' > i' \geqslant i$, for each $i', j' = 1, 2, ..., n + 1$.

These last two properties can be seen from Table 1. It is straightforwardly obtained that *no advantageous merging* holds but there is *advantageous splitting.* For instance, for $n = 3$, $i = 2$ and $j = 3$, then $s_2 = 0.301$ and $s_3 = 0.172$. If these two merge, we have $n = 2$ and $i' = 2$ with $s_{2'} = 0.363$ which is less than the sum of s_2 and s_3. Therefore, there is no advantage from merging. Analogously, we can see that there is an advantage from splitting by moving the problem from $n = 2$ to $n = 3$, i.e., by splitting one of the individuals into two individuals.

Finally, Table 2 resumes these relevant properties and their relationship with our solution. The proofs are omitted because they can be obtained from Proposition 3.

	Universal sharing rule
Anonymity	Yes
Order preservation	Yes
Claims monotonicity	Yes
Resource monotonicity	Yes
Population monotonicity	Yes
No advantageous merging	Yes
No advantageous splitting	No

Table 2: **General properties and the *universal consistent sharing rule.***

6. An example and some comments for applied work

The objective of this section is to provide some guidelines to practitioners. I start with an illustrative example from the river sharing literature (Ambec and Sprumont, 2002; Kilgour and Dinar, 2001). Subsequently, I comment on some issues regarding the estimation and validation of the model.

Consider a river with a given flow. The majority of the river flow is used to sustain the life of animals and plants, and is used for human consumption and economic activities. A remaining extra flow is available, fro example, 10 m^3/s. Two farmers are interested in using this extra flow to expand their productions. Since the flow passes first through Farmer A she can freely consume the full flow leaving no extra flow to Farmer B. However, Farmer B may be able to block Farmer A activity by proceeding to the courts (or through any other means). The outcome of this dispute is time consuming, costly and uncertain. Furthermore, since the property rights over the extra flow are not well-defined it is not even clear that farmer A has any legal obligation because she is not consuming beyond the quantity of water required to keep the function of the society. The described problem has a structure similar to the ultimatum or dictator games, depending on whether or not Farmer B can effectively block Farmer A activities.

Usually, in this type of problems, Farmer A claims for an allocation close to the full resource consumption while Farmer B claims for an allocation close to the equal split. Moreover, usually both parties acknowledge that Farmer A has a better strategic position. Therefore, the solution must consider this aspect. For that reason, there is some agreement that Farmer A should obtain more than half of the extra water flow in order for a negotiated solution to be possible. Otherwise, Farmer A may not accept the terms of the agreement. The objective of the present paper is to offer a solution to this type of problems that has chances of receiving a general consensus.

In this context, if the value of the resource equals 8 billion monetary units, which would correspond to $\lambda = 3$ (on a billions scale), and the society degree or level of altruism is $\alpha = 0.6$, then Farmers A and B should obtain 63.7% and 36.3% of the extra water flow, respectively (see Column (2) of Table 1). In other words, in a river sharing problem over a resource with a value of 8 billion monetary units in a society with a degree or level of altruism of $\alpha = 0.6$, a negotiated solution must award to Farmer A about two times more than to Farmer B.

However, we may also consider situations in which farmers belong to different countries or societies with different values of α and λ. In this case, we should aggregate these values in a unique pair of parameters. There are several aggregation possibilities. For instance, regarding the value of α, the most natural approach is to consider either a simple or a weighted average where the weight given to each country would depend on the populations of both countries. This approach would be more in line with the objective followed in the present paper - the search for a consensual and generally accepted agreement. Regarding the value of the resource, in the case of differences in terms of valuation, the most natural approach is to consider a simple average.

Following Definitions 1 and 2, the conflict between self-interest and equity concerns implies that each individual has a specific degree of altruism. Similarly, for different reasons, each individual has a valuation for the resource under dispute. Therefore, an individual can be characterized by a (α, λ) pair. These variables are not necessarily independent and in some cases their separation can be difficult. Nonetheless, in order to compute Expression (2), we must be able to obtain the aggregate degree of altruism and the expected valuation for the resource.

Usually, the value of the resource can be objectively estimated while the society degree or level of altruism is more subjective and difficult to obtain. In this context, another related question is how to test the allocation proposal in the present paper.

There is a large number of empirical approaches that can deal with these issues. For instance, this theory can be tested through an experiment along the following lines. First, the subjects are asked to answer a set of questions that are designed to reveal their degree or level of altruism and the value of the resource under dispute. Alternatively, or in addition, the subjects can be asked to play either a dictator game or an ultimatum game. Second, the information gathered through this procedure can be used to estimate the sample population degree or level of altruism and the expected value of the resource. Subsequently, in order to obtain an allocation proposal, the obtained parameters can be replaced in Expression (4). Afterwards, the subjects can be asked whether they agree or not with the proposed allocation. Finally, the number of individuals agreeing with the proposed allocation should be contrasted against some election threshold to obtain a measure of the validity of the proposed theory.

The proposed empirical approach seems natural and feasible but is not exclusive. The reader is free to consider alternative approaches.

In addition to river sharing type problems (Kilgour and Dinar, 2001; Ambec and Sprumont, 2002), such as the one described above, the results in the present paper are of interest for several other practical problems. For instance, the n-individuals sequential structure in the present paper can be applied to bargaining problems in which each individual share of the resource is sequentially determined (Curiel et al., 1989; Herings and Predtetchinski, 2012), or to rationing problems in which some individuals hold some sort of priority over others (Moulin, 2000).

Finally, in addition to potential estimation difficulties, a crucial aspect for the implementation of the solution in the present paper is that the involved parties accept ex-ante the proposed allocation rule. In other words, independently of their strategic position, individuals must believe that the obtained result is the most adequate way to divide a resource. This can be challenging because our society may not be ready to accept solutions to practical problems that are not so much based on material considerations, but rather on behavioural aspects.

7. Conclusion

The challenge for conflicting situations between individuals with different bargaining positions is to obtain a self-enforcing and consensual agreement. In order to deal with this difficulty, I propose a practical solution that introduces human behaviour into a resource allocation problem. The behavioural aspects that play a role in a negotiated solution - and their impact - may be difficult to determine. Nonetheless, equity concerns and self-interest seem to be the most salient (Camerer, 2003; Diekmann et al., 1997; Engel, 2011; Ottoni-Wilhelm et al., 2014). These conflicting, but non-contradictory, aspects determine the individual level of altruism. In addition, the value that each individual attaches to the resource also plays a role in the splitting decision. In the present paper, these considerations are aggregated into a sharing rule.

There are some interesting observations and recommendations for further research that can be derived from the results and the approach followed in the present paper. For instance, the results in Proposition 2 suggest that some experiments might be undermined by uninterested subjects that may look altruistic when in fact they do not value enough the resource under dispute. Therefore, we might be overestimating altruistic behaviour. This issue should be subject to further research.

The present paper is a step forward in the introduction of behavioural considerations into resource allocation problems. In this context, there are multiple possibilities in terms of extensions and future work; for instance, the consideration of different theoretical treatments and principles, the imposition of new properties or the relaxation of some others. Independently of the path followed, the future development of the resource allocation literature passes through an increasing consideration of behavioural and psychological aspects that are inherent to the individuals involved in these disputes. This is crucial because they are the ones that ultimately accept or decline the terms of an agreement. Consequently, the shift towards a behavioural focus in operational research seems to be inevitable. Indeed, several authors agree that more research with a behavioural focus is needed. For example, Franco and Hämäläinen (2016) and Hämäläinen et al. (2013) call for considering behavioural factors in the study and practice of operational research; whilst Becker (2016) and Brocklesby (2016) propose a behavioural research agenda and suggest how to implement it - see also the recent special issue on "behavioural Operational Research" published in this journal.

In the context of the present paper, in order for behavioural rules to allocation problems to receive a generalized application, we must observe important changes in the way people deal and resolve disputes. The society must open to behavioural solutions to material problems. While on the one hand, there is a general agreement that behavioural considerations should play an increasing role in our decisions, on the other hand, there is some resistance to accepting and adopting these solutions in real life problems. This can be the greatest challenge to the progress of behavioural allocation rules, in particular, and to behavioural operational research, in general.

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