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legislatures

Marina Bannikova  
Artyom Jelnov

Document de treball n.13- 2016

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Universitat Rovira i Virgili

Facultat d'Economia i Empresa

Av. de la Universitat, 1

43204 Reus

Tel.: +34 977 759 811

Fax: +34 977 758 907

Email: [sde@urv.cat](mailto:sde@urv.cat)

CREIP

[www.urv.cat/creip](http://www.urv.cat/creip)

Universitat Rovira i Virgili

Departament d'Economia

Av. de la Universitat, 1

43204 Reus

Tel.: +34 977 758 936

Email: [creip@urv.cat](mailto:creip@urv.cat)

*Adreçar comentaris al Departament d'Economia / CREIP*

ISSN edició en paper: 1576 - 3382

ISSN edició electrònica: 1988 - 0820

**DEPARTAMENT D'ECONOMIA – CREIP**  
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# The number of parties and decision making in legislatures

Marina Bannikova\*      Artyom Jelnov†

May 2, 2016

## Abstract

This paper proposes a model of a legislature, formed by several parties, which have to vote for or against a certain bill in the presence of a lobbyist who is interested in a certain outcome of the vote. We show that the ease of manipulating a legislature decision by the lobbyist is increasing with the number of parties. A high threshold leads to fewer parties represented, and consequently, decreases the ease of changing a legislature decision by the lobbyist. On the other hand, a high threshold may cause a misrepresentation of voters. We show that if the threshold is higher than 6%, the impact of the misrepresentation effect becomes significant.

## 1 Introduction

By voting for a party, a citizen declares that s/he agrees on its political position. S/He trusts and delegates his voice to this party. Hence, it is not surprising that s/he may be worried about two things. First, whether the party s/he voted for will have a share of the legislature so that his voice will be heard. Second, that if the party gains seats in the legislature, then it will vote sincerely, according to political views that s/he voted for.

The 2015 Catalan (i.e., Catalonia, Spain) general elections gave us an outstanding example of how an electoral threshold can create a misrepresentation in a legislature. Each of 11 parties which participated in the elections were requested to declare their opinion on the matter of Catalonia's independence: two of the parties strictly supported it and five of the parties did not. In the election 48.05% of voters voted for the 'yes' parties and 50.82% voted for 'no' parties. Yet, since one 'no' party and indifferent parties did not pass the electoral threshold of 3%, it appears that 72 MPs (53.33%) belong to 'yes' parties, and 63 MPs (46.66%) to 'no' parties. Therefore, the electoral threshold caused a difference between the opinions of voters and representatives. At the same time, the voters cannot be sure that the parties will vote for or against independence, as they promised. Their vote can change due to the change of their initial position or because of the (il)legal pressure from outside of the parliament.

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\*Universitat Rovira i Virgili and CREIP, Spain, e-mail: marina.bannikova@urv.cat

†Ariel University, Israel, e-mail: artyomj@ariel.ac.il

This situation inspired us to write a model of a legislature, formed by several parties, which have to vote for or against a certain bill. We question what is the effect which the electoral threshold may have on the ease of influencing the voting outcome by a lobbyist through its impact on the number of parties in the legislature. We find that it is easier for a lobbyist to influence on the legislature with more parties. The intuition is that if the number of parties increases, the probability that the lobbyist finds parties that will agree to change their position, also increases. This conclusion is concomitant with some empirical studies (2, 1), who find that countries with a higher number of political parties tend to have higher levels of corruption. More parties in the legislature may be caused by a lower electoral threshold.

We assume that the lobbyist tries to manipulate the legislature decision by suggesting payment to parties for the change of their vote in favour of the vote that is desired by the lobbyist. The minimal demands of parties to change their position are random variables. The maximal total amount of payment that the lobbyist agrees to make, is random, hence s/he needs to solve an optimisation problem: to obtain the simple majority of votes, while minimising his/her total payment to parties. If this total amount is below the maximal total amount the lobbyist agrees to pay, s/he obtains his/her desired decision. Otherwise, parties vote according to their initial standings. We calculate the probability that the final decision by the legislature coincides with the popular will (we call this probability the *fairness* of the political system).

We analyse how this probability depends on the electoral threshold, taking into account both misrepresentation (which increases in the electoral threshold) and ease of manipulation (which increases in the electoral threshold) by the lobbyist effects. Our simulations show that for thresholds that exist in most parliamentary democracies with the proportional representation system (1-5 %), the 'fairness' in our sense is close to being optimal. As for thresholds higher than 6%, 'fairness' is decreasing with the threshold.

## 2 Model and results

### 2.1 Number of parties given

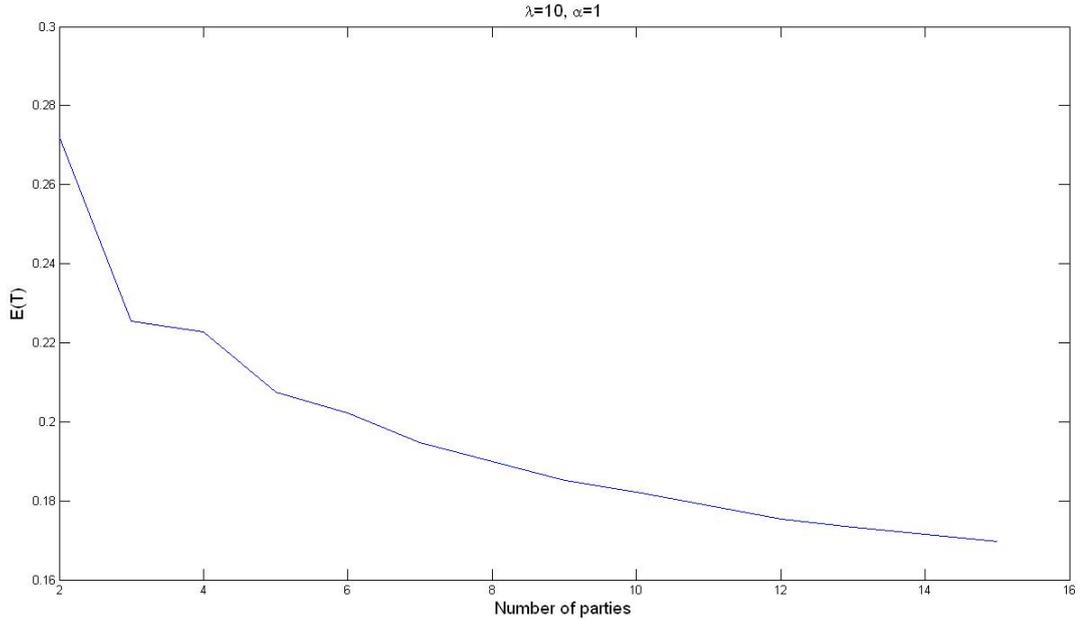
Let  $n$  be the number of parties in a legislature. Let  $D_i \in [-1, 1]$ ,  $i = 1, \dots, n$  be a position of party  $i$  on the bill suggested to be voted for/against. Nonpositive  $D_i$  means that the party  $i$  supports the bill, but will vote against it for payment  $|D_i|$  or more. Positive  $D_i$  means that the party  $i$  opposes the bill, but will vote for it for payment  $D_i$  or more. Assume that positions of parties are random. For each  $i \in \{1, \dots, n\}$

$$D_i = 2X_i - 1 \tag{1}$$

where  $X_i \in [0, 1]$ ,  $X_i \sim Beta(\alpha, \alpha)$ ,  $\alpha$  and  $\lambda$  are given parameters. Note that as  $\alpha \rightarrow \infty$ , the distribution  $Beta(\alpha, \alpha)$  converges to the Bernoulli distribution with parameter  $\frac{1}{2}$ . Namely, as  $\alpha$  increases,  $D_i$ s with increasing probability are closer to 0, which means a less polarised ideologically political system.

We assume that each party has strong party discipline, hence all members of the party cast the same vote. The number of seats in the legislature controlled by each party is its voting weight  $w_i$ , which is assumed to be random:  $w_i \sim Poiss(\lambda)$  for each  $i \in \{1, \dots, n\}$ . A simple majority of votes is required to accept the bill.

Figure 1: Ease of manipulation



Let  $L$  denote the lobbyist. Without loss of generality, we assume that the lobbyist is interested in bill passing.<sup>1</sup> The lobbyist offers to each party a nonnegative payment  $P_i \in [0, 1]$  for voting for the bill, and every party accepts the offer iff

$$P_i - D_i \geq 0$$

$L$  minimises the total amount of payment to parties, subject to being the total weight of parties which support the bill by a simple majority of the total weight of parties in the legislature. Namely, the lobbyist solves

$$T(n) = \min \left\{ \sum_{i=1}^n P_i \right\} \text{ s.t. } \sum_{P_i - D_i \geq 0} w_i > \frac{\sum_{j=1}^n w_j}{2} \quad (2)$$

The expected  $T(n)$  will be denoted as  $E_T(n)$ , which characterises the ease of manipulation the decision of the legislature.

Figure 1 presents the results of a Monte Carlo simulation for  $E_T(n)$  depending on  $n$  (the number of trials is 1,000,000). It shows that the ease of manipulate is increasing with the number of parties, namely, the expected total payment required is decreasing.

## 2.2 Given electoral threshold

Let a random number of parties compete in elections for a given number of seats in a legislature:  $n' \sim Poiss(\lambda_p)$ ,  $\lambda_p$  is a given parameter. Even before the elections all the

<sup>1</sup>If the lobbyist might be for or against the bill with equal probability, the results are same by the way of the symmetry.

parties have their political standings for or against the bill to be voted for after elections:  $D'_i \in [-1, 1]$ ,  $i = 1, \dots, n'$ .  $D'_i$  is randomly picked as in (1).

For  $i = \{1, \dots, n'\}$ , let  $v_i \sim Poiss(\lambda^*)$  be the number of popular votes received by party  $i$ ,  $\lambda^*$  is a given parameter. Assuming that each party position coincides with its voters' position, we define *popular preference*  $\Pi \in \{A, R, I\}$  about the bill as “Accept”, “Reject”, or “Indifferent”:

$$\Pi = \begin{cases} A & , \sum_{D'_i \leq 0} v_i > \sum_{D'_i > 0} v_i \\ R & , \sum_{D'_i \leq 0} v_i < \sum_{D'_i > 0} v_i \\ I & , \sum_{D'_i \leq 0} v_i = \sum_{D'_i > 0} v_i \end{cases}$$

A widely-used institutional rule which is implemented in general elections is the electoral threshold. Parties, which obtained a share of popular vote lower than the predefined threshold do not receive any representation in the legislature. Let the electoral threshold  $t \in \{0, \dots, 1\}$  be given. Party  $i$  passes the threshold  $t$  iff  $v_i > t \sum_{j=1}^{n'} v_j$ . Therefore,

$$n = \left\{ i \in n' \mid v_i > t \sum_{j=1}^{n'} v_j \right\}$$

A mapping  $\pi : \{1, \dots, n\} \rightarrow \{1, \dots, n'\}$  is interpreted as following: if party  $i \in \{1, \dots, n\}$  passed the threshold  $t$ , its index among other parties which passed the threshold is  $\pi^{-1}(i)$ .

Seats are allocated to  $n$  parties proportionally to their popular vote share, where surplus votes are allocated according to the widely-used d'Hont method (see 3). Given  $v_{\pi(1)}, \dots, v_{\pi(n)}$ , let  $w_1, \dots, w_n$  be the number of seats, or weights, allocated to  $n$  parties.

$D_i$  is an initial position of party  $i$ , which passed the threshold, on the bill. Similarly to section 2.1,  $|D_i|$  means the minimal demand for payment to party  $i$  to vote for the bill. Note, that  $D_i \leq 0$  means that party  $i$  supports the bill if there is no payment.

As in section 2.1,  $P_i \in [0, 1]$  is the lobbyist's payment offer to party  $i$ ,  $i \in \{1, \dots, n\}$ , which accepts it iff

$$P_i - D_i \geq 0$$

The lobbyist L's standing  $D_L$  about the bill is randomly chosen as in (1). Negative  $D_L$  means that L supports the bill, positive means that s/he opposes it and  $D_L = 0$  means indifference. L agrees to pay to parties for changing their standing about the bill in an amount less than  $|D_L|$ . If this sum is not sufficient to assure the majority in the legislature desired by the lobbyist, s/he prefers to not make any payment. Let

$$D_i = \begin{cases} D'_{\pi(i)} & , D_L \leq 0 \\ -D'_{\pi(i)} & , D_L > 0 \end{cases}$$

$i = 1, \dots, n$ .

L offers non-negative payments  $P_1, \dots, P_n$  to parties, such that the total payment  $T(n)$  minimises (2). If  $T(n) < |D_L|$ , the legislature adopts the decision  $\Pi_l(t)$  about the bill as desired by the lobbyist, namely,

$$\Pi_l(t) = \begin{cases} A & , D_L < 0 \\ R & , D_L > 0 \\ I & , D_L = 0 \end{cases}$$

In the case  $T(n) \geq |D_L|$ , the lobbyist does not make any payment and parties in the legislature vote according to their initial standings. Namely, for  $T(n) \geq |D_L|$

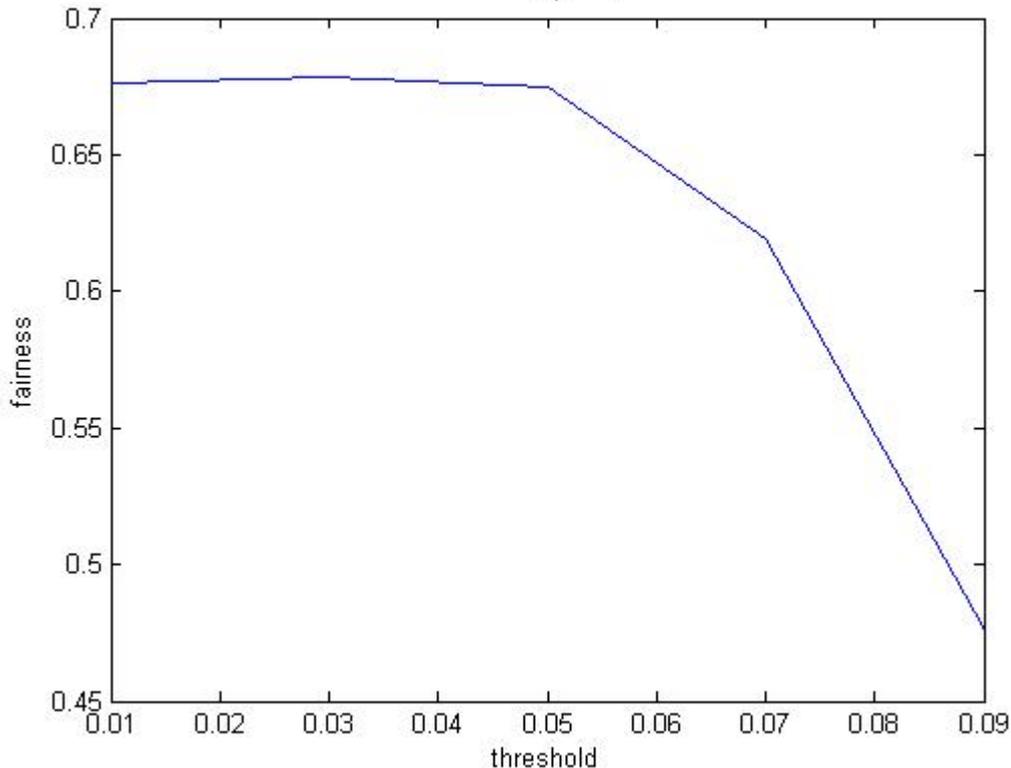
$$\Pi_l(t) = \begin{cases} A & , \sum_{D_i < 0} w_i > \sum_{D_i > 0} w_i \\ R & , \sum_{D_i < 0} w_i < \sum_{D_i > 0} w_i \\ I & , \sum_{D_i < 0} w_i = \sum_{D_i > 0} w_i \end{cases}$$

We define the *fairness* of the threshold  $t$  as the probability that  $\Pi = \Pi_l(t)$ .

Figure 2 presents results for 100 seats, 100,000 trials.

Figure 2: Fairness of threshold

$$\lambda^*=15000, \lambda_p=10, \alpha=1$$



It follows from Figure 2, that for thresholds below 6%, fairness does not suffer; as for thresholds higher than 6%, the 'fairness' is decreasing with the threshold.

**Remark** Results for different values of  $\alpha$  are similar for models in sections 2.1 and 2.2. These models should not be sensitive to changes in the number of seats, in  $\lambda$  and  $\lambda^*$ , since results depend on fraction of seats/popular votes of each party out of a total number of seats/popular votes.

**Acknowledgements.** Financial support by COST Action IC1205 on Computational Social Choice is acknowledged.

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