UNIVERSITAT
ROVIRA I VIRGILI
DEPARTAMENT D'ECONOMIA

(creip)
CENTRE DE RECERCA EN ECONOMIA
INDUSTRIAL I ECONOMIA PÚBLICA

WORKING PAPERS

Col·lecció "DOCUMENTS DE TREBALL DEL DEPARTAMENT D'ECONOMIA - CREIP"

Is the Assumption of Linearity in Factor Models too Strong in Practice?

Nektarios Aslanidis
Luke Hartigan

Document de treball n.3- 2016

DEPARTAMENT D'ECONOMIA – CREIP
Facultat d'Economia i Empresa

UNIVERSITAT
ROVIRA I VIRGILI
DEPARTAMENT D'ECONOMIA

Edita:

Departament d'Economia www.fcee.urv.es/departaments/economia/public_html/index.html Universitat Rovira i Virgili Facultat d'Economia i Empresa Av. de la Universitat, 1 43204 Reus Tel.: +34 977 759 811 Fax: +34 977 758 907 Email: sde@urv.cat	CREIP www.urv.cat/creip Universitat Rovira i Virgili Departament d'Economia Av. de la Universitat, 1 43204 Reus Tel.: +34 977 758 936 Email: creip@urv.cat

Adreçar comentaris al Departament d'Economia / CREIP

ISSN edició en paper: 1576 - 3382
ISSN edició electrònica: 1988 - 0820

DEPARTAMENT D'ECONOMIA – CREIP
Facultat d'Economia i Empresa

Is the Assumption of Linearity in Factor Models too Strong in Practice?

Nektarios Aslanidis[1]
Universitat Rovira i Virgili, CREIP

Luke Hartigan
University of New South Wales

March 21, 2016

Abstract

The assumption of linearity of factor models is implicit in all empirical applications used in macroeconomic analysis. We test this assumption in a more general setting than previously considered using a well-studied macroeconomic dataset on the U.S. economy, and find strong evidence in support for regime-switching type non-linearity. Furthermore, we show non-linearity is strongly concentrated in certain groups (such as financial variables). Our results, which are robust to serial dependence, suggest the assumption of linearity underpinning factor models might be too strong and gives further support towards developing models which explicitly account for non-linearity.

Keywords: Factor Model Non-linearity, Regime Change, Transition Variables, LM test.

JEL Classifications: C12; C18; C24; C33; C38.

---

[1] This research was initiated during 2015-16 while the first author was visiting the University of New South Wales, and the hospitality of the School of Economics during his stay is gratefully acknowledged.
Correspondence to: nektarios.aslanidis@urv.cat.

1. Introduction

An important assumption implicit in all factor analysis of macroeconomic data is that of linearity of the underlying factor model. However, this assumption of linearity is at odds with broad evidence of temporal variation in many macroeconomic forecasting relations (Stock and Watson, 1996). Much recent empirical work has highlighted the existence of non-linearities in factor models when used in empirical applications. Non-linearity is shown to exist in both the factor loadings (Stock and Watson, 2009, Breitung and Eickmeier, 2011, Chen et al., 2014, and Han and Inoue, 2016) as well as in the processes describing the factors themselves (Hartigan, 2015).

Motivated by the literature on the Great Moderation, Stock and Watson (2009) find evidence of an abrupt break in the factor loadings when considering the break date of March 1984 (based upon Chow-type tests). Their result was further reinforced by the works of Breitung and Eickmeier (2011) and Yamamoto (2016), although both use a slightly different data set. On the other hand, Chen et al. (2014) proposed a two-stage procedure to detect breaks in factor loadings by testing the parameter instability in a regression of the first principal component over the remaining principal components. Their results suggest a break in the factor loadings around 1979-1980 (which corresponds to the Iranian revolution at the beginning of 1979 and its subsequent impact on global energy prices and U.S. inflation), slightly earlier than the date suggested by previous authors. Han and Inoue (2016) introduced a test for structural breaks in factor loadings based on the second moments of the estimated factors. In contrast, evidence in Hartigan (2015) indicates that changes in the processes describing the factors are associated with peaks and troughs of the business cycle[2], and once this is accounted for the author finds moderate support for regime changes in the factor loadings, although less than in previous studies. The main conclusion gained from these previous studies is that models with constant factor loadings will fail to properly take account of changes in an economy over time.

However, in light of these findings, the issue of whether the linearity assumption underlying factor modelling of macroeconomic data remains valid has not been explicitly

[2]It is worth mentioning that the possibility of changes in the factor processes was also investigated by Han and Inoue (2016) in an earlier working version of their paper.

investigated to date. More specifically, previous tests of constancy of factor loadings have focused solely on discrete breaks, presumably resulting from short-term events such as a recession or some other type of transitory shock. Longer term changes such as the transition from manufacturing-based economies to more services-based economies (as has been the case in many developed economies such as the U.S. in the period following WWII), which will involve some gradual change in the inter-relationships between different sections of the economy, might be missed by these same tests. Our contribution relative to the existing literature is that we directly test for non-linearity using a more general framework. This method allows for both abrupt breaks as well as more gradual changes in an approximate factor model of the U.S. economy using the well-known Stock and Watson (2005) dataset (hence forth SW2005). To achieve this we adopt the linearity test proposed by Luukkonen et al. (1988) and popularized in the macro-econometrics literature by Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993) and van Dijk et al. (2002). This setting is well suited for our purposes as the model under the alternative is a smooth transition regime-switching type specification.

Our results indicate strong evidence for general regime-switching type non-linearity. In comparison, previous studies have provided evidence of only a one-time change in the factor loadings (Stock and Watson, 2009, Breitung and Eickmeier, 2011, and Chen et al., 2014). Our analysis also shows strongest support for non-linearity when using interest-rate related series as a transition variable. Since changes in factor loadings relate to changes in the correlation structure of the underlying data, this indicates that interest rate variables provide important information about changes in the inter-relationships between different sections of an economy over time. From a forecasting perspective this result is intuitive since interest rates are often used as leading indicators of economic activity (see Stock and Watson (2003) for an autoregressive distributed lag (ADL) case, and Galvão (2006) for a threshold vector autoregression (VAR) case).

Moreover, we find that between 41-47% of the outlier adjusted SW2005 panel and between 43-58% of the raw SW2005 data set rejects the null hypothesis of linearity at a 1% significance level when using between six and eight common factors. This is solid evidence against linearity. For instance, using outlier-adjusted data Stock and Watson (2009) finds that while 41% of their panel rejects constant factor loadings using a

5% significance level, at the 1% level they find only 23% of the series reject the null of a structural break. The fraction of rejections in Breitung and Eickmeier (2011) although seemingly higher than ours, (between 48-55% for the outlier-adjusted data, and 61-67% for the raw data), use a significance level of 5%. Nonetheless, our results are comparable to Breitung and Eickmeier (2011) when using 5% significance level (we find rejection rates of 54-60% and 56-65% for the outlier-adjusted data and raw data, respectively). Another important result drawn from our study is that non-linearity is more prevalent in certain groups of series such as financials, housing and money/credit than in others such as production, employment and prices. As a robustness check, we use both standard OLS estimation and feasible two-step GLS estimation (allowing for serially correlated errors) and our results are generally consistent for both estimation methods. Indeed, the tests based on feasible two-step GLS estimation indicate even stronger non-linearity compared to the other studies.

We conclude that the assumption of linearity in Approximate Factor models might be too strong when used with macroeconomic data and gives further support towards models which explicitly account for non-linearity as a new avenue for research.

The paper is organized as follows: Section 2 presents the econometric methodology and the test for non-linearity, while Section 3 provides the empirical results. Finally, Section 4 briefly concludes.

2. Econometric Framework

2.1 The Smooth Transition Factor Model

Factor models decompose the covariation of observable economic variables $y_{i,t}$, $i=1,...,N$, $t=1,...,T$ into the sum of two unobservable components; one that affects all $y_{i,t}$ s, namely the common factors, and one that is idiosyncratic (unique to each *i*). In practice, we work with the static representation of the approximate factor model:

$$y_{i,t}=\lambda'_i F_t+\varepsilon_{i,t} \tag{1}$$

where $F_t=(F_{1,t},...,F_{r,t})'$ is an *r*-dimensional vector of common factors, where *r* is "small" ($r << N$), λ_i is the corresponding vector of *r* factor loadings, and $\varepsilon_{i,t}$ denotes the idiosyncratic component. Under fairly weak assumptions, the factors and factor loadings can be estimated consistently by principal components as $N,T \to \infty$ (Stock and Watson, 2002, and Bai and Ng, 2002).

We consider a regime-switching extension of model (1) with a possibly smooth transition in the factor loadings. The idea of smooth transition in regression coefficients is not unusual and was initially proposed in the macro-econometrics literature by Teräsvirta and his co-authors. The process describing the smooth transition factor model is defined as:

$$y_{i,t} = \lambda'_i F_t + (\theta'_i F_t) G_{i,t}(s_{i,t-1};\gamma_i,c_i) + \varepsilon_{i,t} \tag{2}$$

where $G_{i,t}(s_{i,t-1};\gamma_i,c_i)$ denotes a series-specific transition function ($i = 1,...,N$), which changes smoothly from 0 to 1 as $s_{i,t-1}$ increases. Then the factor loadings themselves change smoothly between λ_i and $(\lambda_i+\theta_i)$. To see this more clearly, consider the logistic function as the transition function given by:

$$G_{i,t}(s_{i,t-1};\gamma_i,c_i) = \left(1+\exp(-\gamma_i(s_{i,t-1}-c_i))\right)^{-1},\ \gamma_i > 0 \tag{3}$$

where $s_{i,t-1}$ denotes the transition variable, and the slope parameter γ_i indicates how smoothly the switch from 0 to 1 is while c_i is the location parameter (determines where the switch occurs). Although this setup allows for gradual change, it can also accommodate more abrupt changes in factor loadings. For instance, when $\gamma_i \to \infty$, $G_{i,t}(s_{i,t-1};\gamma_i,c_i)$ becomes a step function, and the switch between the regimes is abrupt. In that case, the model approaches a threshold model in factor loadings.

2.2 Testing for Linearity

Testing linearity in factor loadings illustrates the so-called 'unidentified nuisance parameters' problem in the sense that more than one set of restrictions can be used to make the non-linear factor model collapse to a linear one. Besides $H_0{:}\theta_i = 0$, the null of linearity can alternatively be expressed as $H'_0{:}\gamma_i = 0$. The main consequence of the presence of nuisance parameters is that conventional statistical theory is not available for obtaining the asymptotic null distribution of the test statistics.

Among the different solutions to this issue suggested in the literature, the method by Luukkonen et al. (1988) is the most commonly used in the smooth transition regression modelling. More specifically, the authors proposed to replace the transition function by its Taylor series approximation (e.g., they use a third-order approximation) around $\gamma_i = 0$. In the re-parameterized equation, the identification problem is no longer present, and linearity can be tested by means of a Lagrange Multiplier (LM)-type test. The test, to be referred as the LM_1 linearity test in our work, is obtained from the auxiliary regression:

$$y_{i,t} = \lambda'_i F_t + \beta'_{i,1}(F_t s_{i,t-1}) + \beta'_{i,2}(F_t s^2_{i,t-1}) + \beta'_{i,3}(F_t s^3_{i,t-1}) + \varepsilon^*_{i,t} \tag{4}$$

For a given transition variable, we test for the overall null hypothesis $H_0 : \beta_{i,1} = \beta_{i,2} = \beta_{i,3} = 0$ ($i = 1,..., N$). As long as $\sqrt{T} / N \to 0$, one can ignore factor estimation error and treat the factors as 'data' (see Bai and Ng, 2006), and therefore standard asymptotic inference can be used to test the null hypothesis that Eq. (4) is linear in the loadings (parameters). Thus a LM test is performed with a standard χ^2 distribution and degrees of freedom set equal to the number of restrictions imposed.

One could argue that since the proposed testing procedure circumvents the identification problem by an appropriate linearisation, information about the non-linear structure under the alternative is lost and the power may be adversely affected. However, Skalin (1998) investigated this issue by constructing a parametric bootstrap likelihood ratio (LR) test of linearity against the smooth transition regression alternative. The author found that although it has good size properties, the bootstrap LR test is generally less powerful than the auxiliary regression based test of Luukkonen et al. (1988). Furthermore, Luukkonen et al. (1988) showed their test has good power properties even when the regime switch is abrupt (when the model is threshold model). Therefore, we proceed with the use of this test.

In addition, we assume that the transition variable is unknown and implement a general linearity test as in Luukkonen et al. (1988). This test, to be referred as the LM_2 linearity test is obtained from the auxiliary regression:

$$y_{i,t} = \lambda'_i F_t + \sum_{k=1}^{r} \sum_{j=k}^{r} \beta_{i,1kj} F_{k,t} F_{j,t} + \sum_{j=1}^{r} \beta_{i,3j} F^3_{j,t} + \varepsilon^*_{i,t} \tag{5}$$

We test for the significance of the squared, cubed terms and cross products of the factors. Thus, the null hypothesis is given by $H_0{:}\beta_{i,1kj}{=}\beta_{i,3j}{=}\,0$ ($i=1,\ldots,N$), for all k, j.

3. Empirical Results

3.1 Baseline Results

We apply our two tests to the SW2005 data set, which is a well-studied macroeconomic data set. While the data set is now relatively old, using it insures our results are comparable to previous results in the factor modelling literature (see for example, Bai and Ng, 2002 and Breitung and Eickmeier, 2011, Hartigan 2015, and Yamamoto, 2016). The data set contains 132 monthly U.S. economic series and the sampling period runs from 1959:1 to 2003:12. Each series is transformed by removing outliers and taking logs and/or differencing so that the transformed series are approximately stationary (for details, we refer to Stock and Watson, 2005)[3]. We call this data set the 'outlier-adjusted' set[4]. For comparison, we also use another dataset without removing outliers, termed 'raw'. The Bai and Ng (2002) IC_{p2} criterion suggests that the number of common factors in the data set is $\hat{r}=7$ and $\hat{r}=8$ for the outlier-adjusted and raw data, respectively (as in Stock and Watson, 2005). Hence, for robustness we checked results with $6-8$ factors, but for the LM_1 test to conserve space we focused on 7 factors only.

When implementing the LM_1 test, we aimed to be as agnostic as possible about selecting the most appropriate transition variable(s), and therefore we examined all series in the panel. In particular, for each of the 132 series in the SW2005 dataset we performed the LM_1 test by using each one of these variables as the candidate transition variable, and reported the fraction of rejections at the 1% level (note, the candidate transition variable is lagged by 1 month to avoid any simultaneity problems). Table 1 presents results for some key macroeconomic variables acting as the transition variable while results using other macroeconomic variables of importance for policy makers are reported in Tables A1-A6 in the Appendix. We also categorize the SW2005 panel into six

[3] The data is obtained from http://www.princeton.edu/~mwatson/.

[4] We treated the outliers exactly as Stock and Watson (2005) did which was to remove any observation greater 6 times the inter-quantile range.

subcategories according to their qualitative nature (a similar classification is used in Yamamoto, 2016 and Hartigan, 2015), and report linearity test results[5]. One objective of this exercise is to find the main sources of non-linearity (that is, which transition variables imply non-linearity most frequently for the rest of the series in the panel); another related objective is to ascertain if these transition variables also show the strongest evidence of non-linearity when used as the dependent variable in our test.

Several conclusions can be drawn from the LM_1 test results. Firstly, interest rates appear to be the dominant source of regime-switching type non-linearity in the SW2005 dataset. When interest rate-related series act as the transition variable the test results show a very high proportion of rejections of linearity for the total number of variables (see the last row in Table A1). For example, the rejection rates are 71% in the case of the 3-month Treasury bill rate, 74% in the case of the commercial paper rate and 55% for the Federal Funds rate, while there are fewer rejections when using interest rate spreads (to the Federal Funds rate) as transition variables. Secondly, housing, stock market and money-related series deliver substantially lower rejection rates at about 30-50% (see the last row in Table A2). Thirdly, the rejections rates implied by price-related series can vary considerably across transition variables; for example, 55% when the implicit price deflator acts as the transition variable, but only 22% for the producer price index (see the last row in Table A3). Production, consumption, inventories, as well as employment and orders-related series in Table A4 and Table A5, respectively, show even lower rejection rates at about 7-27%. For the raw data, in most cases the rejection rates increase somewhat**,** but the results remain qualitatively similar. It thus appears that some of the non-linearities in the raw data may arise from outliers.

It is also interesting to compare the results across subcategories. When interest rate-related series appear as the transition variable the test indicates widespread non-linearity confirming the previous result that interest rates are the main source of non-linearity in the SW2005 panel. For example, when using the 3-month Treasury bill rate as the transition variable the rejection rates range from 55% for money/credit (D) to 96% for financials (E). In contrast, when production, consumption or employment-related series act as the transition variable, the rejection rates drop significantly for all subcategories.

[5] The summary names of the six categories are shown in Table 2.

Another important result drawn from this test is that regime-switching type non-linearity is more heavily concentrated in certain groups of series than in others. For example, housing (C) and particularly financials (E) are generally the two groups with the highest proportion of rejections across different transition variables, although it is worth noting that these two categories have a relatively small number of members (see Table 2).

Furthermore, these findings related to interest rates appear to correspond to the actual estimated factors themselves. Hartigan (2015) documents that the second, third and fifth factors from the SW2005 data set seem to be mostly associated to interest rate spreads (the second factor) and interest rates (factors three and five), while factors six and seven appear mostly linked to housing and stock-market variables, respectively. We confirm this result in Table 3, which provides summary statistics for the estimated R-squared coefficient from a sequence of regressions of each of the first seven factors on each series in the SW2005 data set separately. It is clear that interest rate-related variables seem to have a significant impact on the other variables in this particular data set on the U.S. economy.

The question of whether there is evidence of regime-switching type non-linearity when the transition variable is unknown is examined by the LM_2 test results in Table 4. On the basis of 6-8 factors and with outlier adjustment, our results indicate that between 41-48% of overall panel rejects the null of linearity at 1% significance (see the last row). The rejection rates for the raw data increase slightly at around 43-54%, which is in agreement with the LM_1 test results. Overall, this is strong evidence against linearity compared to results in previous research. For instance, using outlier-adjusted data Stock and Watson (2009) finds that while 41% of their panel rejects parameter constancy using a 5% significance level, at the 1% level they find only 23% of the series reject the null of a structural break. The fraction of rejections in Breitung and Eickmeier (2011) although seemingly higher than ours (48-55% for outlier-adjusted data, and 61-67% for raw data), use a significance level of 5%. Yamamoto (2016), who builds on the work of these authors, finds that around 65% of the SW2005 data set have 'unstable' factor loading when using his preferred testing procedure. Nonetheless, our results are comparable to both Breitung and Eickmeier (2011) and Yamamoto (2016) when using 5% significance level. For example, we find rejection rates of 54-60% and 56-65% for the outlier-adjusted and raw data, respectively (see the last row in Table A6). More importantly, the results in Stock and Watson (2009) and Breitung and Eickmeier (2011) provide evidence of only a one-time discrete shift in the factor loadings, while our results provide evidence of non-linearity in the factor loadings in general.

Furthermore, compared to previous studies, our method is more able to identify the main source(s) of non-linearity which those others studies are unable to do. For instance, Table 4 illustrates that non-linearity is more widespread in certain groups of series than in others as was the case with the LM_1 in Table 1. Evidently, for financials (E), linearity is rejected overwhelmingly (77-88% for the outlier-adjusted data, and 81-92% of the raw data) followed by housing (C) (80% rejection rate across factors and raw vs. outlier-adjusted data). Interestingly, both Hartigan (2015) and Yamamoto (2016) document similar findings in relation to these two subcategories. There is also strong evidence of non-linearity for money/credit (D), although the rejections rates drop substantially for the outlier-adjusted data (from between 55-73% to 18-36%). As before, there are relatively fewer rejections for production, employment and price-related variables.

3.2 Additional results and robustness checks

It is plausible to think the results in previous section may overstate non-linearity due to possible serial correlation in the data. To assess whether this is the case we compute the LM_1 and LM_2 tests by using a feasible two-step GLS estimation (as in Breitung and Eickmeier, 2011). Specifically, the factor loadings in the auxiliary test regressions are estimated by taking into account possible serial correlation in the idiosyncratic component. Here our primary focus is on the LM_2 test obtained from the following auxiliary regression:

$$(y_{i,t}-\rho_i y_{i,t-1}) = \lambda'_i(F_t-\rho_i F_{t-1}) + \sum_{k=1}^{r}\sum_{j=k}^{r}\beta_{i,1kj}(F_{k,t}-\rho_i F_{k,t-1})(F_{j,t}-\rho_i F_{j,t-1}) + \sum_{j=1}^{r}\beta_{i,3j}(F_{j,t}^3-\rho_i F_{j,t-1}^3) + \tilde{\varepsilon}_{i,t}^* \quad (6)$$

allowing the idiosyncratic errors in Eq. (1) to follow individual-specific $AR(1)$ processes. To obtain estimates of ρ_i, we first run the OLS regressions of $\hat{\varepsilon}_{i,t}$ on $\hat{\varepsilon}_{i,t-1}$, where $\hat{\varepsilon}_{i,t}$ is the principal component estimator of the idiosyncratic component.

Table 5, which is similar to Table 4, displays the results from this alternative estimation. For the total number of series (see the last row), the results largely confirm the baseline findings with the rejection rates remaining high. Notably, based on 7 factors the rejection rate is 41% for the outlier-adjusted data and 55% for the raw data compared to baseline rejection rates of 42% and 52%, respectively. More importantly, when using a significance level of 5% (and on the basis of 6-8 factors) we find rejection rates of 63-64% and 63-67% for the outlier-adjusted and raw data, respectively (see the last row in Table A7). These rejection rates are indeed very strong. For example, when using the HAC version of their test Breitung and Eickmeier (2011) report rejections rates of 50-62% for outlier-adjusted data, and of 61-65% for raw data using 5% level.

Inspection of other parts of Table 3 reveals that the rejection rates subcategories such as financials (E) are still high; for example, 69-81% for the outlier-adjusted data, and 69-77% for the raw data. On the other hand, there are fewer rejections for housing (C) compared to the OLS estimation. Interestingly, linearity is now more often rejected for prices (F) and for money/credit series (D) although mainly for the raw data.

Our results raise some issues for the large dimensional factor literature. For instance, Stock and Watson (2009) argue that if factor loading instability is *mild* and sufficiently *independent* across constituent variables, then the use of a large number of series in the estimation of the factors can average out such instability. However, we show that factor loading non-linearity is rather strong and seemingly concentrated of certain groups (such as financial series). Overall, we conclude that the above results challenge the assumption of linearity implicit in factor models of the U.S. economy and give further support towards developing models which *explicitly* account for non-linearity as a new avenue for research.

4. Conclusions

Factor models provide an efficient way to summarize information from large dimensional economic data sets and have received extensive attention in the macro-econometrics literature over many decades. However, implicit in their use in empirical applications is the assumption the model is linear in factor loadings. Using two alternative tests and a well-studied macroeconomic dataset for the U.S. economy, we have provided statistical evidence that suggests the assumption of linearity is potentially too strong in practice.

This finding has important empirical and theoretical implications. For example, much of the asymptotic theory underpinning the use of factor models is built upon the assumption of linearity; if this assumption does not hold in empirical settings, then it suggests new theory which explicitly allows for non-linearities might be needed. Finally, our results provide further support towards developing alternative factor models which explicitly account for non-linearity as a new avenue for research.

Table 1: LM_1 linearity for given transition variable

(Proportion of rejections at 1% significance level)

Outlier-adjusted data

Category / Transition variable	IPS10	CES002	A0M057	HSFR	A0M070	PMNO	FSPCOM	FYFF	PWFSA	FM2
A	0.18	0.18	0.23	0.28	0.05	0.08	0.15	0.36	0.18	0.18
B	0.36	0.12	0.28	0.32	0.20	0.20	0.04	0.56	0.52	0.04
C	0.30	0.50	0.30	0.70	0.30	0.80	0.90	0.80	0.00	0.10
D	0.00	0.00	0.00	0.18	0.18	0.37	0.18	0.46	0.09	0.55
E	0.00	0.12	0.00	0.27	0.27	0.62	0.04	0.81	0.15	0.04
F	0.00	0.00	0.00	0.00	0.00	0.24	0.05	0.52	0.19	0.43
Total	0.14	0.14	0.14	0.27	0.14	0.31	0.15	0.55	0.22	0.19

Raw data

Category / Transition variable	IPS10	CES002	A0M057	HSFR	A0M070	PMNO	FSPCOM	FYFF	PWFSA	FM2
A	0.08	0.23	0.56	0.21	0.08	0.18	0.10	0.87	0.28	0.23
B	0.20	0.12	0.48	0.08	0.24	0.24	0.16	0.92	0.68	0.32
C	0.30	0.30	0.90	0.80	0.20	0.50	0.60	0.60	0.60	0.70
D	0.00	0.00	0.55	0.18	0.09	0.55	0.09	0.64	0.09	0.82
E	0.27	0.15	0.04	0.50	0.27	0.77	0.58	0.92	0.35	0.39
F	0.05	0.05	0.48	0.10	0.00	0.29	0.29	0.81	0.62	0.52
Total	0.14	0.15	0.46	0.27	0.14	0.38	0.27	0.84	0.43	0.41

Notes: Selected transition variables.

IPS10: INDUSTRIAL PRODUCTION INDEX-TOTAL INDEX (Out)
CES002: EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE (EMP)
A0M057: Manufacturing and trade sales (RTS)
HSFR: HOUSING STARTS (HSS)
A0M070: Manufacturing and trade inventories (Inv)
PMNO: NAPM NEW ORDERS INDEX (Ord)
FSPCOM: S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (Spr)
FYFF: INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (Int)
PWFSA: PRODUCER PRICE INDEX (Pri)
FM2: MONEY STOCK: M2 (Mon)

Table 2: Categories defined as in Yamamoto (2016)

Category		Count
A	*Income / Consumption / Employment*	*39*
B	*Production / New orders / Inventories*	*25*
C	*Housing*	*10*
D	*Money / Credit*	*11*
E	*Stock price / Interest Rates / Exchange Rates*	*26*
F	*Consumer Price / Producer Price*	*21*
	Total	*132*

Table 3: R-squared values for factors: Summary statistics

Factors	*Mean*	*Std. Dev.*	*Maximum*	*Variable*
F1	0.173	0.207	0.741	CES003
F2	0.071	0.128	0.651	sFYBAAC
F3	0.052	0.070	0.283	FYGT5
F4	0.049	0.149	0.717	GMDCN
F5	0.042	0.060	0.239	FYGT5
F6	0.034	0.064	0.332	HSBSOU
F7	0.027	0.069	0.459	FSPIN

Notes: This table shows summary statistics of the estimated R-squared values from a sequence of regressions of each of the seven factors on each series in the SW2005 data set separately.

CES003: EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING (EMP)
sFYBAAC: sFYBAAC - FYFF (Int)
FYGT5: INTEREST RATE: U.S. TREASURY, 5-YR. (Int)
GMDCN: PCE, IMPL PR DEFL: PCE; NONDURABLES (Pri)
HSBSOU: HOUSES AUTHORIZED: SOUTH(THOU.U.) S.A. (HSS)
FSPIN: S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (Spr)

Table 4: LM_2 linearity for unknown transition variable-test regression by *OLS*
(Proportion of rejections at 1% significance level)

	Outlier-adjusted data			Raw data		
Category / # of Factors	$\hat{r}=6$	$\hat{r}=7$	$\hat{r}=8$	$\hat{r}=6$	$\hat{r}=7$	$\hat{r}=8$
A Income / Consumption / Employment	0.23	0.18	0.31	0.28	0.36	0.36
B Production / New orders / Inventories	0.36	0.40	0.40	0.24	0.36	0.36
C Housing	0.80	0.80	0.80	0.80	0.80	0.80
D Money / Credit	0.18	0.27	0.36	0.55	0.73	0.73
E Stock price / Interest / Exchange Rates	0.77	0.85	0.88	0.81	0.88	0.92
F Consumer Price / Producer Price	0.29	0.29	0.33	0.24	0.33	0.38
Total	0.41	0.42	0.48	0.43	0.52	0.54

Table 5: LM_2 linearity for unknown transition variable- test regression by feasible *GLS*
(Proportion of rejections at 1% significance level)

	Outlier-adjusted data			Raw data		
Category / # of Factors	$\hat{r}=6$	$\hat{r}=7$	$\hat{r}=8$	$\hat{r}=6$	$\hat{r}=7$	$\hat{r}=8$
A Income / Consumption / Employment	0.41	0.28	0.41	0.46	0.49	0.46
B Production / New orders / Inventories	0.32	0.24	0.36	0.40	0.36	0.28
C Housing	0.70	0.50	0.20	0.60	0.40	0.20
D Money / Credit	0.18	0.09	0.27	0.64	0.73	0.73
E Stock price / Interest / Exchange Rates	0.69	0.85	0.81	0.69	0.69	0.77
F Consumer Price / Producer Price	0.48	0.43	0.48	0.67	0.67	0.67
Total	0.46	0.41	0.46	0.55	0.55	0.52

Bibliography

Bai J. and S. Ng (2002), Determining the number of factors in approximate factor models, *Econometrica* **70,** 191-221.

Bai J. and S. Ng (2006), Confidence intervals for diffusion index forecasts with a large number of predictors, *Econometrica* **74,** 1133-1150.

Breitung J. and S. Eickmeier (2011), Testing for structural breaks in dynamic factor models, *Journal of Econometrics* **163**, 71-84.

Chen L., J.J. Dolado and J. Gonzalo (2014), Detecting big structural breaks in large factor models, *Journal of Econometrics* **180**, 30-48.

Galvão A.B. (2006), Structural break threshold VARs for predicting US recessions using the spread, *Journal of Applied Econometrics* **21,** 463–487.

Granger C.W.J. and T. Teräsvirta (1993), *Modelling nonlinear economic relationships*, Oxford: Oxford University Press.

Hamilton J. (1989), A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica* **57,** 357-384.

Han X. and A. Inoue (2016), Tests of parameter instability in dynamic factor models, forthcoming in *Econometric Theory*.

Hartigan L. (2015), Changes in the factor structure of the US economy: Permanent breaks or business cycle regimes? UNSW Business School Research Paper No. ECON17.

Luukkonen R., P. Saikkonen and T. Terasvirta (1988), Testing linearity against smooth transition autoregression, *Biometrika* **75,** 491-499.

Skalin J. (1998), Modelling macroeconomic time series with smooth transition autoregressions, Ph.D. Thesis, Stockholm School of Economics.

Stock J.H. and M.W. Watson (1996), Evidence on structural instability in macroeconomic time series relations, *Journal of Business and Economic Statistics* **14,** 11-30.

Stock J.H. and M.W. Watson (2003), Forecasting output and inflation: The role of asset prices, *Journal of Economic Literature* **41,** 788-829.

Stock J.H. and M.W. Watson (2009), Forecasting in dynamic factor models subject to structural instability. In J. Castle and N. Shephard (Eds.), The Methodology and Practice of Econometrics: A Festschrift in Honour of David F. Hendry, pp. 173-205. Oxford University Press.

Teräsvirta T. and H.M. Anderson (1992), Characterizing nonlinearities in business cycles using smooth transition autoregressive models, *Journal of Applied Econometrics* **7,** S119-S136.

van Dijk D., T. Teräsvirta and P.H. Franses (2002), Smooth transition autoregressive models- a survey of recent developments, *Econometrics Reviews* **21,** 1-47.

Yamamoto Y. (2016), Forecasting with non-spurious factors in U.S. macroeconomic timer series, *Journal of Business and Economic Statistics* **34,** 81-106.

Appendix

Table A1: LM_1 linearity for given transition variable-Interest rates (Int)
(Proportion of rejections at 1% significance level)

Outlier-adjusted data

Category / Transition variable	FYFF	CP90	FYGM3	FYGT10	FYAAAC	FYBAAC	scp90	sfygm3	sFYGT10	sFYAAAC
A	0.36	0.54	0.59	0.49	0.26	0.39	0.28	0.49	0.46	0.13
B	0.56	0.72	0.88	0.68	0.40	0.52	0.24	0.36	0.68	0.16
C	0.80	1.00	0.80	0.50	0.50	0.60	0.90	1.00	1.00	0.60
D	0.46	0.46	0.55	0.46	0.46	0.37	0.36	0.46	0.36	0.09
E	0.81	0.92	0.96	0.85	0.81	0.65	0.58	0.96	0.58	0.42
F	0.52	0.76	0.62	0.33	0.43	0.33	0.14	0.29	0.05	0.14
Total	0.55	0.71	0.74	0.57	0.46	0.47	0.36	0.56	0.49	0.23

Raw data

Category / Transition variable	FYFF	CP90	FYGM3	FYGT10	FYAAAC	FYBAAC	scp90	sfygm3	sFYGT10	sFYAAAC
A	0.87	0.87	0.80	0.82	0.64	0.41	0.36	0.49	0.28	0.18
B	0.92	0.92	0.84	0.88	0.68	0.52	0.12	0.32	0.12	0.16
C	0.60	0.90	0.90	0.90	0.30	1.00	0.90	1.00	0.90	0.70
D	0.64	0.91	0.73	0.73	0.46	0.28	0.28	0.18	0.27	0.18
E	0.92	0.92	1.00	0.96	0.89	0.73	0.54	0.89	0.39	0.50
F	0.81	0.86	0.95	0.67	0.48	0.43	0.14	0.38	0.00	0.14
Total	0.84	0.89	0.87	0.83	0.63	0.53	0.35	0.53	0.27	0.27

Notes: Selected transition variables.

FYFF: INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (Int)
CP90: Commercial Paper Rate (Int)
FYGM3: INTEREST RATE: U.S.TREASURY BILLS, 3-MO. (Int)
FYGT10: INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR. (Int)
FYAAAC: BOND YIELD: MOODY'S AAA CORPORATE (Int)
FYBAAC: BOND YIELD: MOODY'S BAA CORPORATE (Int)
scp90: CP90- FYFF (Int)
sfygm3: FYGM3- FYFF Int)
sFYGT10: FYGT10- FYFF (Int)
sFYAAAC: FYBAAC - FYFF (Int)

Table A2: LM_1 linearity for given transition variable-Housing starts (HSS), Stock prices (SPr), Money and credit quantity aggregates (Mon)

(Proportion of rejections at 1% significance level)

Outlier-adjusted data

Category / Transition variable	HSFR	HSBR	PMI	FSPCOM	FSDXP	FM1	FM2	FM3	FMRNBA	FCLBMC
A	0.28	0.23	0.21	0.15	0.36	0.13	0.18	0.10	0.10	0.46
B	0.32	0.16	0.28	0.04	0.60	0.04	0.04	0.08	0.08	0.48
C	0.70	0.40	0.50	0.90	1.00	0.20	0.10	0.00	0.20	0.90
D	0.18	0.18	0.18	0.18	0.36	0.18	0.55	0.46	0.55	0.46
E	0.27	0.19	0.42	0.04	0.77	0.08	0.04	0.15	0.50	0.31
F	0.00	0.19	0.43	0.05	0.48	0.00	0.43	0.29	0.33	0.19
Total	0.27	0.21	0.32	0.15	0.55	0.09	0.19	0.16	0.26	0.42

Raw data

Category / Transition variable	HSFR	HSBR	PMI	FSPCOM	FSDXP	FM1	FM2	FM3	FMRNBA	FCLBMC
A	0.21	0.31	0.28	0.10	0.21	0.13	0.23	0.23	0.62	0.36
B	0.08	0.44	0.28	0.16	0.40	0.08	0.32	0.20	0.88	0.16
C	0.80	0.90	0.60	0.60	0.90	0.10	0.70	0.00	1.00	0.70
D	0.18	0.27	0.18	0.09	0.18	0.46	0.82	0.82	0.91	0.27
E	0.50	0.46	0.35	0.58	0.96	0.31	0.39	0.54	0.96	0.15
F	0.10	0.38	0.43	0.29	0.43	0.10	0.52	0.52	0.76	0.14
Total	0.27	0.42	0.33	0.27	0.48	0.17	0.41	0.36	0.81	0.27

Notes: Selected transition variables.

HSFR: HOUSING STARTS (HSS)
HSBR: HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (HSS)
PMI: PURCHASING MANAGERS' INDEX (HSS)
FSPCOM: S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (Spr)
FSDXP: S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (Spr)
FM1: MONEY STOCK: M1 (Mon)
FM2: MONEY STOCK: M2 (Mon)
FM3: MONEY STOCK: M3 (Mon)
FMRNBA: DEPOSITORY INST RESERVES (Mon)
FCLBMC: WKLY RP LG COM'L BANKS (Mon)

Table A3: LM_1 linearity for given transition variable-Price Indexes (Pri)
(Proportion of rejections at 1% significance level)

Outlier-adjusted data									
Category / Transition variable									
	PWFSA	PSCCOM	PSM99Q	PMCP	PUNEW	PUXF	PUXHS	PUXM	GMDC
A	0.18	0.18	0.03	0.28	0.15	0.18	0.03	0.03	0.59
B	0.52	0.16	0.08	0.56	0.20	0.24	0.08	0.20	0.48
C	0.00	0.00	0.00	0.10	0.40	0.10	0.10	0.10	0.00
D	0.09	0.27	0.18	0.09	0.18	0.36	0.00	0.00	0.46
E	0.15	0.39	0.08	0.31	0.58	0.39	0.39	0.46	0.65
F	0.19	0.19	0.10	0.05	0.57	0.52	0.24	0.19	0.76
Total	0.22	0.21	0.07	0.27	0.33	0.30	0.14	0.18	0.55

Raw data									
Category / Transition variable									
	PWFSA	PSCCOM	PSM99Q	PMCP	PUNEW	PUXF	PUXHS	PUXM	GMDC
A	0.28	0.26	0.03	0.39	0.10	0.36	0.00	0.18	0.56
B	0.68	0.32	0.04	0.64	0.20	0.32	0.08	0.48	0.60
C	0.60	0.40	0.00	0.20	0.10	0.60	0.00	0.00	0.80
D	0.09	0.55	0.36	0.18	0.09	0.46	0.00	0.09	0.73
E	0.35	0.46	0.23	0.31	0.23	0.69	0.00	0.08	0.58
F	0.62	0.67	0.10	0.05	0.24	0.81	0.05	0.14	0.76
Total	0.43	0.41	0.11	0.33	0.17	0.52	0.02	0.19	0.64

Notes: Selected transition variables.

PWFSA: PRODUCER PRICE INDEX (Pri)
PSCCOM: SPOT MARKET PRICE INDEX: ALL COMMODITIES (Pri)
PSM99Q: INDEX OF SENSITIVE MATERIALS PRICES (Pri)
PMCP: NAPM COMMODITY PRICES INDEX (Pri)
PUNEW: CPI-U: ALL ITEMS (Pri)
PUXF: CPI-U: ALL ITEMS LESS FOOD (Pri)
PUXHS: CPI-U: ALL ITEMS LESS SHELTER (Pri)
PUXM: CPI-U: ALL ITEMS LESS MEDICAL CARE (Pri)
GMDC: PCE, IMPL PR DEFL: PCE (Pri)

Table A4: LM_1 linearity for given transition variable-Output and income (Out), Consumption (PCE), Inventories and inventory-sales ratios (Inv)
(Proportion of rejections at 1% significance level)

Outlier-adjusted data

Category / Transition variable	a0m052	A0M051	IPS10	IPS11	IPS25	IPS43	A0m082	A0M224_R	PMNV	A0M070
A	0.08	0.13	0.18	0.05	0.05	0.10	0.10	0.08	0.18	0.05
B	0.00	0.00	0.36	0.00	0.16	0.32	0.28	0.08	0.36	0.20
C	0.00	0.00	0.30	0.00	0.00	0.00	0.10	0.20	0.70	0.30
D	0.18	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.18
E	0.08	0.00	0.00	0.00	0.08	0.04	0.04	0.00	0.27	0.27
F	0.10	0.10	0.00	0.00	0.00	0.05	0.05	0.00	0.19	0.00
Total	0.07	0.06	0.14	0.15	0.06	0.11	0.11	0.07	0.27	0.14

Raw data

Category / Transition variable	a0m052	A0M051	IPS10	IPS11	IPS25	IPS43	A0m082	A0M224_R	PMNV	A0M070
A	0.08	0.10	0.08	0.10	0.10	0.10	0.10	0.21	0.15	0.08
B	0.00	0.00	0.20	0.08	0.32	0.16	0.04	0.08	0.28	0.24
C	0.00	0.00	0.30	0.10	0.00	0.50	0.50	0.30	0.50	0.20
D	0.09	0.09	0.00	0.00	0.00	0.18	0.09	0.00	0.09	0.09
E	0.19	0.15	0.27	0.00	0.04	0.42	0.58	0.00	0.12	0.27
F	0.05	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.29	0.00
Total	0.08	0.08	0.14	0.05	0.10	0.20	0.20	0.10	0.21	0.14

Notes: Selected transition variables.

a0m052: Personal income (Out)
A0M051: Personal income less transfer payments (Out)
IPS10: INDUSTRIAL PRODUCTION INDEX-TOTAL INDEX (Out)
IPS11: INDUSTRIAL PRODUCTION INDEX-PRODUCTS (Out)
IPS25: INDUSTRIAL PRODUCTION INDEX-BUSINESS EQUIPMENT (Out)
IPS43: INDUSTRIAL PRODUCTION INDEX-MANUFACTURING (Out)
A0m082: Capacity Utilization (Out)
A0M224_R: Real Consumption (PCE)
PMNV: NAPM INVENTORIES INDEX (Inv)
A0M070: Manufacturing and trade inventories (Inv)

Table A5: LM_1 linearity for given transition variable-Employment and hours (EMP), Orders and unfilled orders (Ord)

(Proportion of rejections at 1% significance level)

Outlier-adjusted data										
Category / Transition variable										
	LHEL	LHEM	LHNAG	LHUR	A0M005	CES002	A0M048	CES155	A0M008	A1M092
A	0.03	0.15	0.10	0.10	0.05	0.18	0.28	0.13	0.00	0.15
B	0.00	0.08	0.00	0.16	0.04	0.12	0.32	0.04	0.00	0.28
C	0.00	0.10	0.10	0.20	0.00	0.50	0.80	0.00	0.10	0.20
D	0.09	0.00	0.18	0.00	0.27	0.00	0.27	0.00	0.00	0.09
E	0.00	0.04	0.04	0.04	0.08	0.12	0.39	0.04	0.00	0.19
F	0.00	0.05	0.05	0.05	0.00	0.00	0.05	0.00	0.00	0.00
Total	0.02	0.08	0.07	0.09	0.06	0.14	0.31	0.05	0.08	0.16

Raw data										
Category / Transition variable										
	LHEL	LHEM	LHNAG	LHUR	A0M005	CES002	A0M048	CES155	A0M008	A1M092
A	0.05	0.18	0.10	0.18	0.10	0.23	0.23	0.05	0.03	0.08
B	0.00	0.00	0.00	0.08	0.04	0.12	0.32	0.08	0.04	0.28
C	0.00	0.00	0.00	0.30	0.00	0.30	0.80	0.00	0.20	0.10
D	0.09	0.00	0.18	0.00	0.27	0.00	0.18	0.00	0.00	0.09
E	0.00	0.00	0.00	0.15	0.00	0.15	0.39	0.04	0.12	0.15
F	0.05	0.05	0.05	0.05	0.00	0.05	0.10	0.05	0.00	0.00
Total	0.03	0.06	0.05	0.13	0.06	0.15	0.30	0.05	0.05	0.12

Notes: Selected transition variables.

LHEL: INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (EMP)
LHEM: CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (EMP)
LHNAG: CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (EMP)
LHUR: UNEMPLOYMENT RATE (EMP)
A0M005: Average weekly initial claims, unemploy. insurance (EMP)
CES002: EMPLOYEES ON NONFARM PAYROLLS (EMP)
A0M048: Employee hours in nonag. establishments (EMP)
CES155: AVERAGE WEEKLY HOURS OF PRODUCTION (EMP)
A0M008: Mfrs' new orders, consumer goods and materials (Ord)
A1M092: Mfrs' unfilled orders, durable goods indus (Ord)

Table A6: LM_2 linearity for unknown transition variable-test regression by *OLS*
(Proportion of rejections at 5% significance level)

	Outlier-adjusted data			Raw data		
Category / # of Factors	$\hat{r}=6$	$\hat{r}=7$	$\hat{r}=8$	$\hat{r}=6$	$\hat{r}=7$	$\hat{r}=8$
A Income / Consumption / Employment	0.49	0.48	0.54	0.51	0.59	0.54
B Production / New orders / Inventories	0.40	0.40	0.52	0.36	0.44	0.56
C Housing	0.90	0.90	0.90	0.90	0.90	0.90
D Money / Credit	0.36	0.36	0.36	0.55	0.73	0.73
E Stock price / Interest / Exchange Rates	0.81	0.92	0.89	0.92	0.92	0.96
F Consumer Price / Producer Price	0.38	0.43	0.43	0.29	0.48	0.43
Total	0.54	0.57	0.60	0.56	0.64	0.65

Table A7: LM_2 linearity for unknown transition variable- test regression by feasible *GLS*
(Proportion of rejections at 5% significance level)

	Outlier-adjusted data			Raw data		
Category / # of Factors	$\hat{r}=6$	$\hat{r}=7$	$\hat{r}=8$	$\hat{r}=6$	$\hat{r}=7$	$\hat{r}=8$
A Income / Consumption / Employment	0.64	0.54	0.64	0.54	0.64	0.59
B Production / New orders / Inventories	0.52	0.44	0.48	0.60	0.52	0.48
C Housing	0.70	0.60	0.70	0.70	0.80	0.70
D Money / Credit	0.36	0.36	0.46	0.64	0.82	0.82
E Stock price / Interest / Exchange Rates	0.73	0.89	0.89	0.69	0.77	0.89
F Consumer Price / Producer Price	0.71	0.67	0.57	0.71	0.76	0.71
Total	0.63	0.60	0.64	0.63	0.69	0.67