Market Transparency, Market Quality and Sunshine Trading

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Abstract

This paper analyzes the implications of pre-trade transparency on market performance. We find that transparency increases the precision held by agents, however we show that this increase in precision may not be due to prices themselves. In competitive markets, transparency increases market liquidity and reduces price volatility, whereas these results may not hold under imperfect competition. More importantly, market depth and volatility might be positively related with proper priors. Moreover, we study the incentives for liquidity traders to engage in sunshine trading. We obtain that the choice of sunshine/dark trading for a noise trader is independent of his order size, being the traders with higher liquidity needs more interested in sunshine trading, as long as this practice is desirable.

Key words: Market Microstructure, Transparency, Prior Information, Market Quality, Sunshine Trading

1 Introduction

One of the most surprising phenomena in the microstructure of financial markets is the heterogeneity in pre-trade transparency exhibited by different trading venues.\(^1\) Although one could argue that most of the modern stock trading platforms distribute information on depth, accessible to traders either by subscribing

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\(^1\)Pre-trade transparency refers to the wide dissemination of price quotations and orders before trade takes place.
directly to the market feed, or by purchasing a consolidated feed, it is also true that in the last ten years there has been a tendency to introduce anonymity into stock, bond, and foreign exchange markets.\(^2\) Similarly, we are nowadays envisioning the evolution to dark trading in exchange markets.\(^3\) An investigation on dark trading can be found in Bloomfield et al. (2011). They compare visible markets in which all orders must be displayed, Iceberg (or reserve) markets that allow both displayed and partially displayed orders, and Hidden markets in which orders can be non-displayed.\(^4\)

There is broad agreement that transparency matters; it affects the informativeness of the order flow and, hence, the process of price discovery, but the key effects of transparency on security markets are complex and contradictory. As pointed out by Eom et al. (2007) “...there is no consensus on whether an increase in pre-trade transparency results in an improvement or deterioration in market quality.” These authors study changes in pre-trade transparency in the Korea Exchange. They conclude that market quality is increasing in pre-trade transparency. In the same line Boehmer et al. (2005) find that the introduction of OpenBook by the NYSE leads to a more active management of trading strategies and improvements in terms of liquidity and informational efficiency. These results contrast with findings derived in Madhavan et al. (2005). This paper shows that an increase in pre-trade transparency in the Toronto Stock Exchange leads to wider spreads, lower depth, and higher volatility. This is consistent with the empirical evidence from the French Stock Exchange where liquidity increased after anonymity was introduced (see Foucault et al. (2007)), the same occurred when brokers identification codes were removed at The Tokyo Stock Exchange (see Comerton-Forde et al. (2005)). Similarly, the experiments by Bloomfield et al. (2011) support the robustness of informational efficiency and liquidity in opaque regimes too.

In this article we are concerned with pre-trade transparency in the form of disclosing information about the composition of the order flow to market participants. Depending of who takes the disclosure decision, two types of pre-trade transparency can be distinguished: mandatory/prohibited and voluntary. In the former the decision whether to reveal (or not to reveal) information about the composition of the order flow is taken by the exchange. In the later, the investors voluntarily decide whether to reveal the orders.

One of the studies focused on the implications of the first type of pre-trade transparency is Madhavan (1996). He compares two trading mechanisms, called opaque and transparent. In the opaque market the exchange does not reveal

\(^2\)Anonymity was introduced into the French Stock Exchange in 2001 and into the Italian one in 2004. See Rindi (2008) for a comparison of anonymous versus pre-trade transparent regimes.

\(^3\)Dark pools are trading systems where there is no pre-trade transparency of orders in the system. They can be split into two types: systems such as crossing networks in which cross orders are not subject to pre-trade transparency requirements, and trading venues, such as regulated markets and MTFs, that use waivers for avoiding to display orders. By contrast, lit markets are pre-trade transparent.

\(^4\)Other issues related to pre-trade transparency are the comparison of floor versus automated trading systems and the logic for trading halts, among others.
any information about the composition of the order flow, whereas in the transparent market the exchange reveals the price insensitive component that comes from traders who have liquidity needs. He shows that there exists an inverse relationship between price volatility and market depth; for some parameter configurations an increase in transparency delivers the desirable properties of higher liquidity and lower price volatility, whereas for others it can exacerbate volatility and decrease liquidity. These results are derived under the proviso that rational investors hold improper or non-informative priors about the liquidation value of the risky asset.\footnote{Bayesians believe only in proper priors as a foundation for statistics. For a discussion on the relationship between proper and improper prior distributions, see O'Hagan (1994).} We here propose to frame Madhavan’s analysis in a more canonical way. We assume that rational investors are endowed with proper priors. The assumption of proper priors is more realistic to us, as in financial markets traders negotiate regularly and, therefore, ex-ante they are not completely ignorant about the liquidation value of the assets they trade.

We show that in competitive markets, transparency increases market liquidity and reduces price volatility, whereas under imperfect competition the implications of market transparency are ambiguous. More importantly, the inverse relationship between price volatility and market liquidity, obtained in competitive markets or when investors have improper priors, may not hold with imperfect competition and proper priors, i.e., the inverse relationship between market depth and price volatility reported in Madhavan (1996) may not hold if investors have proper priors.\footnote{Although we lack of a clear-cut intuition for the need of uninformative priors to deliver Madhavan’s results, we believe that the arguments behind them are along the following lines. The use of improper priors is equivalent to the use of no prior information. By contrast, informative priors convey information on what are the relevant values of the parameters when it comes to study the properties that a given model satisfies.} If market depth and volatility are negatively related, an increase in transparency either stabilizes prices and increases market liquidity (both of them suitable properties of a financial market) or increases volatility and reduces liquidity (both of them undesirable for a market). Since preferences for both of these market indicators are aligned, one could conclude that transparency is unambiguously a good or a bad property for a market to have. However, if such a (negative) relation does not hold then there are trade-offs among these two market indicators, and a clear ranking between transparent and opaque markets may not exist.

Additionally, we find that transparency increases the precision of traders’ predictions about the liquidation value. However, the comparison on market liquidity across market structures does also depend on prior specification as with proper priors the opaque market is deeper for a larger parameter specification set.

This paper also addresses the issue of voluntary disclosure of the orders, prior to trading, of some liquidity traders to the other participants, a practice known as sunshine trading. We study the incentives for liquidity traders to engage in sunshine trading. We obtain that the choice of sunshine/dark trading for a noise trader is independent of his order size. Moreover, as long as sunshine
trading is desirable, the traders with higher liquidity find more profitable this practice.

Sunshine trading has also been analyzed by Admati and Pfeiderer (1991). They find that the identification of liquidity orders reduces the trading costs of those who preannounce, but its effects on the trading costs and welfare of other traders are ambiguous. They also show that sunshine trading increases the informativeness of the price whereas it reduces the variance of the price change. Admati and Pfeiderer consider a continuum of informed agents, who are price-takers and their motive for trading is information. We here consider a finite number of informed traders, who behave strategically and their motive for trading is information and hedging. Thus, our results assess the impact of relaxing the assumption of a continuum of informed agents in their conclusions. Another difference between Admati and Pfeiderer’s paper and ours is that we endogenize the choice of the preannouncement.

The remainder of this paper is organized as follows. Section 2 outlines the notation and the hypotheses of the model. Section 3 characterizes the unique symmetric linear equilibrium in a general framework. Section 4 examines the implications of transparency when the disclosure decision is taken by the exchange and Section 5 deals with the choice of sunshine/dark trading by individuals. Section 6 provides some concluding comments. Finally, proofs are relegated to the Appendix.

2 The Model

We consider a pure exchange economy where at time 0 a risky asset is traded in an auction market against a riskless bond, whose return is normalized to zero. The liquidation value of the risky asset in period 1 is represented by the random variable \( \bar{v} \). We assume that \( \bar{v} \) is normally distributed with finite mean \( \bar{v} \) and variance \( \sigma_v^2 \). The risky asset is traded at the automatic market-clearing price \( p \), so that its return is given by its liquidation value minus the price.

In this economy there are \( N \) strategic traders indexed by \( n \) who are endowed with assets and are privately informed about the liquidation value of the risky asset. Trader \( n \) holds an initial endowment of \( y_n \) shares of the riskless asset and an initial endowment of \( \omega_n \) shares of the risky asset. Traders have CARA preferences over final wealth of the form \( U(W_n) = -\exp(-\rho W_n) \), where \( \rho > 0 \) denotes the common coefficient of risk aversion and \( W_n \) represents the final wealth.

\(^7\)Dia and Pouget (2011) finds evidence of sunshine trading in the West-African Bourse.

\(^8\)As in most models of market microstructure (see for instance, Hellwig (1980), Diamond and Verrechia (1981), Glosten and Milgrom (1985), Kyle (1985, 1989), Huddart et al. (2001), among others) we suppose that priors are proper. By doing so, we are able to determine how the strength of prior information affects the robustness of the results on market metrics provided by Madhavan (1996). Moreover, we will try to recover his results, whenever possible, by taking a large variance of \( \bar{v} \). The use of limits to recover the results under improper priors is standard. See, for instance, Morris and Shin (2003) in the context of global games.
wealth for the $n$-th investor, which is given by

$$\tilde{W}_n = (\bar{v} - p) q_n + \bar{v} \omega_n + y_n,$$

where $q_n$ denotes the units of the risky asset traded by investor $n$. Thus, traders are risk averse and they choose their desired order quantities to hedge their risk exposures. But portfolio hedging is not their sole motive for trade, as strategic traders possess private information. Before trading takes place, each strategic trader receives a private signal conveying information about the liquidation value of the risky asset. Let $\hat{s}_n = v + \bar{v} n$ denote the private signal of the $n$-th trader, where $v$ is the realized value of the risky asset and $\bar{v} n$ is a random variable which represents the private signal error for the $n$-th investor. Traders use their information to speculate in the market so that part of trade is information-motivated with transaction volume arising endogenously. In addition to the demands of the speculative traders, there is another component of order flow, denoted by $\tilde{z}$, which represents the imbalances arising from uninformed or noisy traders whose demands are price inelastic. We assume that $\tilde{z}$ is independent of the remainder random variables of the model.

Informed traders submit their (net) demand functions to a unique auctioneer, who aggregates all the demand schedules and the noisy demand and calculates the price at which the market clears. At this price, the auctioneer allocates quantities to satisfy traders’ demands.

We end the model assuming that all the random variables $\tilde{\xi}_1, \ldots, \tilde{\xi}_N, \tilde{\omega}_1, \ldots, \tilde{\omega}_N$ are uncorrelated with $\bar{v}$, and normally and independently distributed with a mean normalized to zero and a variance equal to $\sigma^2_{\tilde{\xi}}$ for $\zeta = \tilde{\xi}, \omega$. Finally, their joint distribution is assumed common knowledge.

In this set-up, we will analyze several trading mechanisms that differ in their degree of transparency regarding the random imbalance of uninformed liquidity traders. The first mechanism is opaque in that rational traders do not receive any information about the composition of the order flow. In this framework, we assume that $\tilde{z}$ is normally distributed with a mean normalized to zero and a variance equal to $\sigma^2_{\tilde{z}}$. The second one is transparent so that the realization of $\tilde{z}$ is disclosed to all market participants prior to trading. In this setup the expected value of $\tilde{z}$ is the realization of this random variable, $z$, and its variance is null. Finally, we examine a trading mechanism with the possibility of sunshine trading, i.e., a framework where some liquidity traders voluntarily preannounce the size of their orders.

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9The derivation of this expression is provided in the Appendix (see computations previous to Equation (15)).

10Throughout the paper, given the random variable $\tilde{x}$, we will denote its realized value by $x$.

11Models assuming a CARA utility function plus a normal distribution of all random variables abound in the market microstructure literature. Moreover, they are the most commonly used even nowadays, as, for instance, in Rindi (2008).
3 The Symmetric Linear Equilibrium in a General Framework

In order to compute the optimal demand schedules in the aforementioned trading protocols, we define and characterize a symmetric linear equilibrium in a more general framework, assuming that the expected value of the noise demand, denoted by \( \overline{z} \), may be not null. Then, giving values to the expected value and the variance of the noise demand, we will obtain the characterizations of the symmetric linear equilibria for the different trading mechanisms. For instance, in the opaque market \( \overline{z} = 0 \) and \( \sigma^2_z > 0 \), whereas in the transparent market \( \overline{z} = z \) and \( \sigma^2_z = 0 \).

Let \( q_n(\cdot; I_n) \) denote the net demand schedule of informed trader \( n \), given that he has observed the information set \( I_n \), where \( I_n = (s_n, \omega_n, y_n) \), i.e., \( I_n \) contains the private signals observed by trader \( n \). Thus, \( q_n = q_n(p; I_n) \) will represent the net quantity demanded by the \( n \)-th informed investor, for particular realizations of both the price of the risky asset \( \tilde{p} \) and the vector which collects his information \( I_n \).

In this economy, we will search for a rational expectations equilibrium under imperfect competition (REE, for short) in which traders follow identical linear demand functions. A REE defines a Bayes-Nash equilibrium in demand functions. In words, it is a price and a set of demand schedules such that the market clears and each trader \( n \), given his information set, submits demand orders which maximize his conditional expected utility taking into account the effect of his trading on prices and taking as given the strategies of other traders. We next define it formally.

**Definition 1:** A rational expectations equilibrium is a price \( q \) and a vector of strategies \( q = (q_1, \cdots, q_N) \), such that:

i) Excess demand is zero at the equilibrium price

\[
\sum_{n=1}^{N} q_n(p; I_n) + z = 0.
\]

ii) Each trader \( n \) maximizes the expected utility of final period wealth given the strategies of other traders,

\[
q_n(p; I_n) \in \arg \max_{(q_n)} E (U [(\tilde{v} - p) q_n + \tilde{v}\omega_n + y_n] | p, I_n), \quad \text{for} \ n = 1, \ldots, N.
\]

As mentioned above, for the tractability of the analysis, we will focus on symmetric linear rational expectations equilibrium (SLE, for short).

**Definition 2:** A symmetric linear equilibrium is a rational expectations equilibrium in which traders’ net demand functions are identical linear functions, that is,

\[
q_n(p; I_n) = \mu + \beta s_n - \alpha \omega_n - \gamma p,
\]

\[12\]We would like to thank an anonymous referee for this suggestion.
where \( \mu, \beta, \alpha \) and \( \gamma \) are constants.\(^{13}\)

Following Kyle (1989), in a SLE the optimal demand function for trader \( n \) is given by

\[
q_n(p; I_n) = E(\bar{v}|p, I_n) - p - \rho \text{var}(\bar{v}|p, I_n)\omega_n. \tag{2}
\]

This demand maximizes traders’ utility whenever the second order condition for a maximum holds, or, equivalently, if the inequality below is satisfied\(^{14}\)

\[
\frac{2}{(N-1)\gamma} + \rho \text{var}(\bar{v}|p, I_n) > 0. \tag{3}
\]

To study the existence of a SLE, we first write all coefficients of speculators’ demands as functions of the coefficient \( \alpha \). This coefficient is then characterized as a root of a polynomial. If such a root exists, then one might conclude that a SLE exists. These facts are formally stated in the following results:

**Lemma 1:** If there exists a SLE then

\[
\mu = \frac{2\alpha}{\rho \sigma_e^2 (N - (N - 1) \alpha)} \bar{v} - \frac{N - 2}{N - (N - 1) \alpha} \bar{v}, \tag{4}
\]

\[
\beta = \frac{\alpha}{\rho \sigma_e^2} \quad \text{and} \quad \gamma = \frac{\alpha}{\rho \sigma_e^2} \left(1 + \frac{2\sigma_e^2}{(N - (N - 1) \alpha) \sigma_e^2}\right). \tag{5}
\]

**Lemma 2:** If there exists a SLE, \( \alpha \in \left(0, \frac{N-2}{N-1}\right) \). In addition, this coefficient is a root of a polynomial of third degree \( Q(\alpha) = a\alpha^3 + b\alpha^2 + c\alpha - d \), whose coefficients are given by \( a = (\phi + 1)(N - 1)^2 \), \( b = (N - \phi (N - 2)) (N - 1) \), \( c = (N - 1) \varphi \), and \( d = \varphi (N - 2) \), with \( \phi = \rho^2 \sigma_e^2 \sigma_{\epsilon}^2 \) and \( \varphi = \rho^2 \sigma_e^2 \sigma_{\epsilon}^2 z \).

Lemma 2 shows that \( \alpha \) is a function of \( N \) and of two payoff unrelated components in the trade: a liquidity component due to traders’ endowments of the risky asset as endowments can be thought of as liquidity shocks, \( \phi = \rho^2 \sigma_e^2 \sigma_{\epsilon}^2 \), and a liquidity component due to the presence of noise traders, \( \varphi = \rho^2 \sigma_e^2 \sigma_{\epsilon}^2 z \).

We end this subsection providing a sufficient condition for the existence and uniqueness of a SLE.

**Proposition 1:** When \( \sigma_z^2 \neq 0 \), there exists a unique SLE iff \( N \geq 3 \). By contrast, when \( \sigma_z^2 = 0 \), there exists a unique SLE iff \( N < (N - 2)\phi \).

\(^{13}\)With CARA utility, note that the solution of the optimization problem for investor \( n \) stated in Definition 1 is independent of the endowment of the riskless asset for this trader. This is the reason why we omit \( y_n \) in the expression of the demand function for investor \( n \).

\(^{14}\)A complete derivation of Equations (2) and (3) can be found in the Appendix.

\(^{15}\)A similar decomposition is used by Naik et al. (1999) to characterize the SLE in a model with interdealer trading.
This result shows the existence and uniqueness of the \textit{SLE} under general conditions. When \( \sigma_{z}^{2} \neq 0 \), we only require that \( N \geq 3 \) as in Kyle (1989).\textsuperscript{16} As in his model, if there are only two rational investors and the noise demand is infinitely inelastic, these speculators have "too much" power and they would like to trade a boundless quantity. In addition, Proposition 1 indicates that when \( \sigma_{z}^{2} = 0 \) a \textit{SLE} may fail to exist as existence requires that the liquidity shock via endowments is large enough. The rationale for this inequality is as follows. Whenever the price is not a good estimator of the private information (because either \( \sigma_{z}^{2} \) or \( \sigma_{v}^{2} \) are high), or, alternatively, agents face a high inventory cost from maintaining their initial holdings of the risky asset (the coefficient of risk aversion is high), then they find profitable to participate in the market as \( \phi \) will be large enough. But if traders are well informed and/or there is little endogenous liquidity needs (\( \sigma_{z}^{2}, \sigma_{v}^{2} \) and \( \rho \) are low so that \( \phi \) is also low), then agents are unwilling to reveal their information to others as they find the readjustment of their portfolio very expensive. They prefer to consume their initial endowment and this results in a market breakdown. When this is the case, a \textit{SLE} fails to exist.

\section{4 Transparency and its Implications}

In this section we will analyze and compare the opaque mechanism and the transparent mechanism. Recall that in the first one rational traders do not receive any information about the composition of the order flow, whereas in the second one the realization of the noise demand is disclosed to all market participants prior to trading. In what follows, we will use superscripts \( O \) and \( T \) to refer to the opaque and the transparent markets, respectively.

Recall that in the opaque market \( z = 0 \) and \( \sigma_{z}^{2} > 0 \), whereas in the transparent market \( z = z \) and \( \sigma_{z}^{2} = 0 \). Substituting these values in Lemmas 1, 2 and Proposition 1, we obtain the following results:

\textit{Corollary 1: A SLE in an opaque market exists iff} \( N \geq 3 \). \textit{If it exists, then}

\[ 
\begin{align*}
\mu^{O} &= \frac{2\alpha^{O}}{\rho (N - (N - 1) \alpha^{O}) \sigma_{z}^{2}}, \\
\beta^{O} &= \frac{\alpha^{O}}{\rho \sigma_{v}^{2}} \quad \text{and} \quad \\
\gamma^{O} &= \frac{\alpha^{O}}{\rho \sigma_{z}^{2}} \left( 1 + \frac{2\sigma_{v}^{2}}{(N - (N - 1) \alpha^{O}) \sigma_{z}^{2}} \right),
\end{align*}
\]

\textit{where} \( \alpha^{O} \) \textit{is the unique real root belonging to} \( \left( 0, \frac{N-2}{N-1} \right) \) \textit{of the polynomial of degree three} \( Q(\alpha) \).

\textsuperscript{16}When \( N = 2 \) the polynomial \( Q(\alpha) \), stated in Lemma 2, simplifies to \( Q(\alpha) = (\phi + 1)\alpha^{3} + 2\alpha^{2} + \varphi\alpha \), which lacks of real strictly positive roots.
Corollary 2: A SLE in a transparent market exists iff $N < (N - 2)\phi$. If it exists, then

\[
\mu_T^T = \mu_0^T - \mu_1^T z = \frac{2\alpha^T}{\rho(N - (N - 1)\alpha^T)} \sigma^z_v - \frac{N - 2}{N - 1} - \alpha^T \frac{N - (N - 1)\alpha^T}{N - (N - 1)\alpha^T} z,
\]

\[
\beta^T = \frac{\alpha^T}{\rho^2 v^2}
\]

and

\[
\gamma^T = \frac{\alpha^T}{\rho^2 v^2} \left(1 + \frac{2\sigma^2_v}{(N - (N - 1)\alpha^T)\sigma^2_v}\right),
\]

where $\alpha^T = \frac{\phi(N - 2) - N}{(\sigma + 1)(N - 1)}$.

Remark 1 Using the expression of $\alpha^T$, it follows that the coefficient associated with $z$, i.e., $-\mu_1^T$, is equal to $-\frac{1}{N - 2}$. The negativity of this coefficient indicates that in the transparent market strategic traders place orders that partly accommodate the noise demand.

4.1 Transparency and Equilibrium Comparison

Next, we discuss the impact of transparency on the strategic behavior of investors, as it influences not only existence of equilibrium, but also the price intercept and the slope of traders’ demands.

Regarding existence of equilibrium, note that if a SLE exists for the transparent market, it exists for the opaque market as well. The condition for existence in the former, $N < (N - 2)\phi$, must meet $N > 2$ but, as $N$ is a natural number, then it requires $N \geq 3$. Note that this is the condition for existence in the latter. Thus, trading is more robust in the opaque market than in the transparent market. In other words, transparency may induce a form of market failure.\(^{17}\) Moreover, it is important to point out that the conditions for existence of SLE are the same as the ones derived by Madhavan (1996). This result is not surprising given that under non-informative priors, the existence conditions are independent of $\sigma^2_v$.

As for the shape of the demand functions, transparency has two main effects. An increment in the price of the risky asset makes agents more optimistic about its liquidation value, which leads to a smaller reduction in the individual demands as compared to the opaque market. Second, demands become less sensitive to traders’ liquidity shocks and private signals. The rationale is that in the transparent market, there is a higher need for camouflage which makes demands less sensitive to private information. The next corollary summarizes these results.

Corollary 3: Traders’ demands are less sensitive to both private information and price in a transparent market as

\[
\alpha^T < \alpha^O, \quad \beta^T < \beta^O \quad \text{and} \quad \gamma^T < \gamma^O.
\]

\(^{17}\)This form of market failure refers to the non-existence of the type of equilibrium we focus on, the SLE. For the comparison to be meaningful, we will focus on parameter values that satisfy $N < (N - 2)\phi$. 

9
In sum, transparency reduces endogenous liquidity trading creating less risk sharing \((\alpha^T < \alpha^O)\), makes orders less responsive to private information about the liquidation value \((\beta^T < \beta^O)\) and reduces demands’ price-responsiveness \((\gamma^T < \gamma^O)\).

### 4.2 Transparency and Market Quality

In this subsection we examine the economic implications of pre-trade transparency. Specifically, we analyze the differences between the opaque and the transparent market in terms of some measures of market quality.

- **Informational Efficiency**

  The market microstructure literature provides several sensible measures of the informational efficiency of a market.\(^{18}\) One is the precision of the information held by informed traders, measured by \(\text{var}^{-1}(\overline{\nu}|p,I_n)\). Another one is the informational content of the equilibrium price, which captures the information revealed by prices to uninformed traders, measured by \(\text{var}^{-1}(\overline{\nu}|p)\).

  **Proposition 2:** At the time the trade is made and \(z\) is realized, pre-trade transparency unambiguously increases the precision of the information held by informed traders.

  In the transparent market the information held by agents is more precise than in the opaque market. Next, we show that prices by themselves may not contribute to this greater precision as in equilibrium \(\text{var}^{-1}(\overline{\nu}|p^O)\) might be larger than \(\text{var}^{-1}(\overline{\nu}|p^T)\).

  **Proposition 3:** i) Prices are more informative in the transparent market iff the following inequality holds

  \[
  \frac{\alpha^O}{1 - N\mu_1^T} = \frac{(N + \phi)(N - 2) - N}{\phi(\phi + 1)(N - 1)}.
  \]

  ii) If \(\phi(2 + N^2 - 4N) - N^2 > 0\) holds, or if \(\sigma^2_z\) is low enough, then prices are always more informative in the transparent market.\(^{19}\)

  Notice that from the expressions of the market clearing price, given by

  \[
  \bar{p}^O = \frac{1}{N\gamma^O} \left( N\mu^O + \beta^O \sum_{j=1}^N \bar{s}_j - \alpha^O \sum_{j=1}^N \bar{\omega}_j + \bar{z} \right) \quad \text{and} \quad (7)
  \]

  \[
  \bar{p}^T = \frac{1}{N\gamma^T} \left( N\mu_0^T + \beta^T \sum_{j=1}^N \bar{s}_j - \alpha^T \sum_{j=1}^N \bar{\omega}_j + (1 - N\mu_1^T) \bar{z} \right), \quad (8)
  \]

\(^{18}\)See Kyle (1989) for a thorough discussion of these measures.

\(^{19}\)A particular case in which prices are more informative in the transparent market is when \(N\) converges to infinity, a result consistent with those reported in Admati and Pfeiderer (1991).
it follows that
\[
\begin{align*}
\text{var}^{-1}(\tilde{v}|p^O) &= \text{var}^{-1}\left(\frac{1}{\rho \sigma_z^2} \sum_{j=1}^{N} s_j + \sum_{j=1}^{N} \omega_j + \frac{z}{\alpha^O}\right) \\
\text{var}^{-1}(\tilde{v}|p^T) &= \text{var}^{-1}\left(\frac{1}{\rho \sigma_z^2} \sum_{j=1}^{N} s_j + \sum_{j=1}^{N} \omega_j + \frac{(1 - N \mu^T \alpha^T)}{\alpha^T} z\right).
\end{align*}
\]

As $\tilde{v}$ and $\tilde{z}$ are uncorrelated, prices are more informative in the transparent market if $\alpha^O < \alpha^T/(1 - N \mu^T \alpha^T)$. Transparency has two opposite effects on the informativeness of prices: on the one hand, it reduces endogenous liquidity trading ($\alpha^O > \alpha^T$), making prices less informative. On the other hand, it facilitates that noise trading be accommodated ($\mu^T > 0$), leading prices to be more informative. The dominant effect is in general ambiguous, but there are, at least, two instances at which transparency enhances price informativeness. If $\sigma^2_z$ is small enough, then $\alpha^O$ approaches $\alpha^T$ making the first effect insignificant. As the second effect is independent of $\sigma^2_z$, transparency increases the informational content of prices. Similarly, if $\phi$ is large enough, then transparency increases the informativeness of prices. The first effect is not very significant as $\alpha^O - \alpha^T$ is small when $\phi$ is large. The second effect, despite being small as well, becomes the dominant one.

- Market Liquidity

In order to measure the impact of transparency on market liquidity, we now compare the market depth in the two market structures. Following Kyle (1985), the market depth is defined as the quantity of noise trading required to induce the price of the risky asset to boost by one unit. Formally, market depth is given by
\[
\left(\frac{\partial p}{\partial \tilde{z}}\right)^{-1} = N \gamma^O \quad \text{and} \quad \left(\frac{\partial p^T}{\partial \tilde{z}}\right)^{-1} = \frac{N \gamma^T}{1 - N \mu^T \alpha^T}.
\]

From Equation (2) and the market clearing condition, it follows that
\[
p = \frac{\sum_{n=1}^{N} E(\tilde{v}|h_n^M, s_n) + \left(\frac{1}{(N-1)\gamma^T}\right) z + \left(z - \sum_{j=1}^{N} \omega_n\right) \rho \text{var}(\tilde{v}|h_n^M, s_n)}{N},
\]

with
\[
h_n^M = \beta^M \sum_{j \neq n} s_j - \alpha^M \sum_{j \neq n} \omega_j + \delta_{M=O} z, \quad M = O, T,
\]

where $\delta_{M=O}$ is an indicator function that takes value one if $M = O$ and zero if $M = T$. Thus, noise trading affects market price through three channels captured by the three terms in equation above: an adverse-selection effect, a strategic effect and a risk-bearing effect.

The **adverse-selection effect** is captured by the first term via $h_n^M$. An increase in $z$ increases $h_n^O$ without affecting $h_n^T$ (in the transparent market this effect is absent as noise demand is displayed). Speculator $n$ assumes that an increment might be due to his competitors receiving favorable signals about the payoff of
the risky asset. Each speculator therefore adjusts his forecast upwards which generates a price boost.

The strategic effect, the second term \( z/(N - 1)N\gamma^M \), measures the competitiveness of the market or its size via \( N \), as well as the price sensitivity of strategic traders’ demands. As they are less price-sensitive in the transparent market, this second effect is stronger in the transparent market.\(^{20}\) Finally, there is a risk-bearing effect. The market-clearing price must accommodate for inducing risk-averse speculators to trade. As speculators are better informed in the transparent market, this third effect is more important in the opaque market.

Whenever \( N \) converges to infinity the strategic-behavior effect vanishes. The equilibrium price is unambiguously more sensitive to changes in the noise demand in the opaque market. The transparent market is deeper. This result may no longer hold under imperfect competition.

Two sufficient conditions for obtaining a larger market depth in a transparent market are given below.

**Proposition 4:** If

\[
\frac{\sigma^2_v(1 + \phi)}{N\sigma^2_v} + \left(1 - \phi \frac{(N^2 - 4N + 2)}{N^2}\right) < 0,
\]

or, alternatively, if \( \sigma^2_z \) is sufficiently small, then the transparent market is deeper.

Proposition 4 suggests that the comparison of the market liquidity between the two market structures is ambiguous and depends on the parameter specification. The intuition behind Proposition 4 runs as follows. When \( \sigma^2_z \) is small enough, \( \alpha^O \) is close to \( \alpha^T \), making \( \gamma^O - \gamma^T \) very small, so that transparency increases market depth. Regarding the sufficient condition in (9) its intuition is less clear. When \( N \) converges to infinity it simplifies to \( 1 < \phi \). This inequality coincides with the condition that guarantees the existence of the SLE in very competitive markets. Thus, transparency increases market liquidity if the market is sufficiently competitive.

The next figure displays the differences in liquidity, \( DL \), in terms of \( \sigma^2_z \) for different values of \( N \). The black line corresponds to \( N = 10 \), the red one to \( N = 20 \) and the green one to \( N = 30 \). A negative value of \( DL \) indicates that the transparent market is deeper. Figure 1 illustrates that as \( N \) increases the parameter configurations in which the transparent market is more liquid are higher. In the limit, when \( N \) converges to infinity, the transparent market is

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\(^{20}\)To clarify the role of the strategic-behavior effect, note that under perfect competition the optimal investors’ demands are given by

\[
q_0(p, I_n) = \frac{E(\bar{v}|p, I_n) - p - \rho \text{var}(\bar{v}|p, I_n)\omega_n}{\rho \text{var}(\bar{v}|p, I_n)}
\]

(see, for instance, Diamond and Verecchia (1981)). Using these demands’ expressions and the market clearing condition, it follows that the equilibrium price has only the terms associated with the adverse-selection effect and the risk-bearing effect.
more liquid for all $\sigma_z^2$.

Figure 1. Difference in market liquidity, $DL$, as a function of $\sigma_z^2$.

The ambiguity of the effect of changes in pre-trade transparency on liquidity derived in this work is consistent with the empirical evidence. Madhavan et al. (2005) find that an increase in pre-trade transparency in the Toronto Stock Exchange led to lower depth. This result contrasts with the empirical investigation of pre-trade transparency at the NYSE conducted by Boehmer et al. (2005).

- Volatility

The volatility of equilibrium prices is measured by $\text{var}(\bar{v} - \bar{p}^M)$. Straightforward computations yield\(^{21}\)

$$\bar{v} - \bar{p}^O = \frac{\bar{v} - \mu_O^O}{\gamma^O} - \frac{\beta^O}{\gamma^O} \sum_{j=1}^{N} \frac{\bar{s}_j}{N} + \frac{\alpha^O}{\gamma^O} \sum_{j=1}^{N} \frac{\bar{\omega}_j}{N} - \frac{1}{N\gamma^O} \bar{z} \text{ and}$$

$$\bar{v} - \bar{p}^T = \frac{\bar{v} - \mu_T^T}{\gamma^T} - \frac{\beta^T}{\gamma^T} \sum_{j=1}^{N} \frac{\bar{s}_j}{N} + \frac{\alpha^T}{\gamma^T} \sum_{j=1}^{N} \frac{\bar{\omega}_j}{N} - \frac{1 - N\mu_T^T}{N\gamma^T} \bar{z}. $$

Therefore,

$$\text{var}(\bar{v} - \bar{p}^M) = \left( \frac{\partial \bar{p}^M}{\partial z} \right)^2 \sigma_z^2 + g(\alpha^M, r), \text{ with}$$

\(^{21}\)Formulae below can be easily derived from Expressions (7) and (8).

$$\text{var}(\bar{v} - \bar{p}^M) = \left( \frac{\partial \bar{p}^M}{\partial z} \right)^2 \sigma_z^2 + g(\alpha^M, r), \text{ with}$$

$$g(\alpha^M, r) = \left( 1 - \frac{\beta^M}{\gamma_M} \right) \sigma_v^2 + \left( \frac{\beta^M}{N\gamma_M} \right)^2 N\sigma_z^2 + \left( \frac{\alpha^M}{N\gamma_M} \right)^2 N\sigma_{\omega}^2.$$

\(^{21}\)Formulae below can be easily derived from Expressions (7) and (8).
where \( r \) stands for the quotient \( \sigma^2_v / \sigma^2_x \). Substituting the values of the equilibrium coefficients, \( g(\alpha^M, r) \) simplifies to

\[
g(\alpha^M, r) = \frac{\sigma^2_x}{N} \left( 4Nr + (\phi + 1) (N - \alpha^M (N - 1))^2 \right)
\]

To analyze the difference in price volatility,

\[
DV = \text{var}(\bar{v} - \bar{p}^O) - \text{var}(\bar{v} - \bar{p}^T),
\]

we decompose it into two terms, \( DV_1 \) and \( DV_2 \), with

\[
DV_1 = \left[ \left( \frac{\partial p^O}{\partial \zeta} \right)^2 - \left( \frac{\partial p^T}{\partial \zeta} \right)^2 \right] \sigma^2_x \quad \text{and}
\]

\[
DV_2 = g(\alpha^O, r) - g(\alpha^T, r).
\]

The first term \( DV_1 \) shows that the difference in price volatility partly stems from the difference in market liquidity \( DL \). However, the presence of a second term, \( DV_2 \), indicates that there are other factors affecting the difference in price volatility. Whenever \( DV_2 \) vanishes or it is small enough, then

\[
\text{sign} (DV) = \text{sign} \left[ \left( \frac{\partial p^O}{\partial \zeta} \right)^2 - \left( \frac{\partial p^T}{\partial \zeta} \right)^2 \right]
\]

\[
= -\text{sign} \left[ \left( \frac{\partial p^O}{\partial \zeta} \right)^{-1} - \left( \frac{\partial p^T}{\partial \zeta} \right)^{-1} \right] = -\text{sign} (DL).
\]

Then, an inverse relationship between market depth and price volatility emerges, which indicates that the mechanism with greater market price volatility provides the lower market depth.

As Madhavan pointed out, a framework in which this inverse relationship arises is when priors are uninformative. To derive his result, note that Corollaries 1 and 2 imply that in this case (i.e., when \( \sigma^2_v \) converges to infinity)

\[
\left( \frac{\beta^O}{\gamma^O} \right) = \left( \frac{\beta^T}{\gamma^T} \right) = 1 \quad \text{and} \quad \left( \frac{\alpha^O}{\gamma^O} \right) = \left( \frac{\alpha^T}{\gamma^T} \right) = \rho \sigma^2_x.
\]

Hence, \( g(\alpha^O, r) = g(\alpha^T, r) \) (see Equation (11)) and \( DV_2 = 0 \) follows. Consequently, \( \text{sign}DV = -\text{sign}DL \).

Another set-up in which an inverse relationship between market depth and price volatility emerges is in large markets. Note that

\[
\lim_{N \to \infty} \text{var}(\bar{v} - \bar{p}^O) = \lim_{N \to \infty} \text{var}(\bar{v} - \bar{p}^T) = 0.
\]

Consequently,

\[
\lim_{N \to \infty} \left[ \text{var}(\bar{v} - \bar{p}^O) - \text{var}(\bar{v} - \bar{p}^T) \right] = 0.
\]
Moreover, after some algebra, it is easy to show that $DV_1$ is of the order of $\frac{1}{N^2}$, but appealing to the Mean Value Theorem, $\lim_{N \to \infty} N^2 DV_2 = 0$. Thus, $DV_2$ converges to zero faster than $DV_1$ and the result follows.

The analytical derivation of sufficient conditions on the primitives that guarantee a direct relationship between price volatility and market depth is not easy. However, there are some particular cases where we can ensure that this type of relationship is feasible. For instance, if

$$\frac{\sigma_z^2(1 + \phi)}{N \sigma_v^2} + \left(1 - \phi \frac{(N^2 - 4N + 2)}{N^2}\right) > 0$$

(12)

holds, then there exists a unique value of $\sigma_z^2$, say $\hat{\sigma}_z^2$, such that both markets are equally liquid (see the proof of Proposition 4), i.e., $DV_1 = 0$ and $DV = DV_2$. Thus around $\hat{\sigma}_z^2$ volatility and liquidity are lined up. For completeness, we next include some numerical examples illustrating this point.

Figures 2 and 3 expose a particular case in which (12) holds. In Figure 2 $DL$ is displayed in terms of $\sigma_z^2$ for $N = 10$, $\phi = 2$, $\sigma_v^2 = 10$, $\sigma_v = \frac{1}{2}$ and $\rho = 1$. We obtain that $\hat{\sigma}_z^2 = 3.052$. Thus, when $\sigma_z^2 < 3.052$ the transparent market is more liquid, whereas when $\sigma_z^2 > 3.052$ the opaque is deeper. Figure 3 represents $DV$ as a function of $\sigma_z^2$ for the parameter configuration: $N = 10$, $\phi = 2$, $\sigma_v^2 = 10$, $\sigma_v = \frac{1}{2}$ and $\rho = 1$. One can observe how for low values of $\sigma_z^2$ ($\sigma_z^2 < 4.030$) the price volatility is higher in the opaque market, whereas for $\sigma_z^2 > 4.030$ the opposite holds true. Therefore when $\sigma_z^2 \in (3.052, 4.030)$, the opaque market is both more liquid and volatile.

![Figure 2. DL in terms of $\sigma_z^2$.](image1)

![Figure 3. DV in terms of $\sigma_z^2$.](image2)

Next figures, Figure 4.A and Figure 4.B display the differences in volatility in terms of $\sigma_z^2$ for different values of $N$ (4.A). The black line corresponds to $N = 10$, the red one to $N = 20$ and the green one to $N = 30$ (as $N$ increments the parameter configurations in which the transparent market is less volatile becomes higher). In the limit, when $N$ converges to infinity, the transparent market is less volatile (and more liquid) for all $\sigma_z^2$. In Figure 4.B we plot $DV$ as a function of $\sigma_z^2$ for different values of $\sigma_v^2$. The black line corresponds to $\sigma_v^2 = \frac{1}{2}$.
the red one to $\sigma^2_v = 1$ and the green one to $\sigma^2_v = 5$. This figure shows that as $\sigma^2_v$ increases the parameter configurations in which the transparent market is less volatile is higher. In the limit, when $\sigma^2_v$ converges to infinity, the transparent market is less volatile (and more liquid) for all $\sigma^2_z$.

Finally, all these results are summarized as follows:

**Proposition 5:** Under improper priors there is an inverse relationship between market depth and price volatility. This result may no longer hold if priors are proper unless $N$ is large enough.

## 5 Sunshine Trading

In this section we examine the possibility that some liquidity traders voluntarily preannounce the size of their orders to the other market participants, a practice known as sunshine trading. Thus, the timing of the game will be as follows: first, noise traders decide whether or not to announce their orders sizes, and second, trading takes place. We solve by backward induction and, therefore, initially we assume that the number of noise traders who preannounce is fixed. Formally, suppose that the noise demand comes from $H$ liquidity traders, indexed by $h = 1, ..., H$. Thus, $\tilde{z} = \sum_{h=1}^{H} \tilde{z}_h$, where $\tilde{z}_h$ denotes the demand for noise trader $h$. Let $\tilde{z}_h$, $h = 1, ..., H$, be i.i.d. with $\tilde{z}_h \sim N(0, \sigma^2_{z_H})$. There are two types of liquidity traders, announcers and nonannouncers. The announcers are noise traders who preannounce the size of their trades, whereas the nonannouncers do not. Let $A$ denote the subset of liquidity traders who are announcers. Formally,

$$A = \{ h \in \{1, ..., H\} \text{ such that } h \text{ is an announcer} \}.$$

Let $H_A$ represent the cardinality of this set ($0 \leq H_A \leq H$) and let $z_A$ denote the realization of the aggregate demand of announcers, i.e., $z_A = \sum_{h \in A} \tilde{z}_h$. Notice
that $H_A = 0$ corresponds to a framework similar to the opaque mechanism, whereas $H_A = H$ models a setup analogous to the transparent market. Therefore, for these two extremes values of $H_A$, Corollaries 1 and 2 characterize the SLE, respectively. For intermediate values of $H_A$, replacing $\sigma^2_z$ by $z_A$ and $\frac{H-H_A}{H} \sigma^2_z$ in Lemmas 1, 2 and Proposition 1, we derive the following result:

**Corollary 4:** Suppose that $0 < H_A < H$. A SLE exists iff $N \geq 3$. If it exists, then

\[
\mu = \mu_0 - \mu_1 z_A = \frac{2\alpha}{\rho \sigma^2_z (N - (N - 1) \alpha)} - \frac{\alpha}{N - (N - 1) \alpha} z_A,
\]

\[
\beta = \frac{\alpha}{\rho \sigma^2_z} \quad \text{and}
\]

\[
\gamma = \frac{\alpha}{\rho \sigma^2_z} \left( 1 + \frac{2\sigma^2_z}{(N - (N - 1) \alpha) \sigma^2_z} \right),
\]

where $\alpha$ is the unique real root belonging to \( \left( 0, \frac{N-2}{N-1} \right) \) of the polynomial of degree three $Q(\alpha)$, with $\varphi = \frac{H-H_A}{H} \rho^2 \sigma^2_z \sigma^2_z$.

Note that the equilibrium coefficients depend on the number of announcers. In what follows, in order to emphasize this fact, we will write the equilibrium coefficients $\gamma$ and $\mu_1$ as $\gamma(H_A)$ and $\mu_1(H_A)$.

We now focus on the first stage of the game and we study the incentives for noise traders to engage in sunshine trading. We assume that noise traders take this decision by comparing their conditional expected trading costs. Let $C^{\text{NA}}(z_h, H_A)$ ($C^A(z_h, H_A)$) denote the expected trading costs of a nonannouncer (an announcer), conditional on his trade size $z_h$, when there are $H_A$ announcers. Thus, the noise trader $h$ is willing to preannounce his trade size whenever\(^{22}\)

\[
C^A(z_h, H_A + 1) < C^{\text{NA}}(z_h, H_A). \tag{13}
\]

Then, if (13) holds for all $H_A = 0, \ldots, H - 1$, and for all $z_h, h = 1, \ldots, H$, then there exists an equilibrium in which all the noise traders decide to engage in sunshine trading.

Direct computations yield

\[
C^{\text{NA}}(z_h, H_A) = \frac{1}{N \gamma(H_A)} z_h^2 \quad \text{and}
\]

\[
C^A(z_h, H_A) = \frac{1 - N \mu_1(H_A)}{N \gamma(H_A)} z_h^2.
\]

Let us make some comments:

1. The conditional expected profits of a noise trader are lower if he is an- nouncer, whenever $z_h \neq 0$.\(^{22}\) When $C^A(z_h, H_A + 1) = C^{\text{NA}}(z_h, H_A)$, the noise trader $h$ is indifferent between preannouncing his trade or not. We assume that he breaks the tie by choosing not to preannounce his trade. The preannouncement can carry some fixed costs that are not considered in this paper (for instance, the costs of transmitting the information to the exchange).
2. Noise traders with higher liquidity needs will be more interested in sunshine trading as long as this practice is desirable.

3. The choice of preannouncement for a noise trader is independent of his order size. This property is crucial since it implies that a noise trader who has already preannounced his order does not regret this decision when he observes that other noise traders also display their orders.

4. Suppose that $z_h \neq 0$. Note that

$$C^A(z_h, H_A + 1) < C^{NA}(z_h, H_A)$$

if and only if

$$\frac{1 - N\mu_1(H_A + 1)}{N\gamma(H_A + 1)} < \frac{1}{N\gamma(H_A)}.$$  

This inequality shows that the preannouncement has two opposite effects on conditional expected trading costs. First, it reduces the price responsiveness of traders demands ($\gamma(H_A + 1) < \gamma(H_A)$), leading to higher conditional expected trading costs. Second, it facilitates that the order is partly accommodated ($\mu_1(H_A + 1) > 0$) leading to lower conditional expected trading costs. Whenever the second effect dominates, the noise trader $h$ will wish to become an announcer.

5. In the Appendix it is shown that whenever $z_h \neq 0$, $C^{NA}(z_h, H_A)$ is increasing in $H_A$, whereas the shape of $C^A(z_h, H_A)$ depends on the parameter configuration.

Whenever $C^A(z_h, H_A)$ is decreasing (and this property is satisfied when $\phi$ is high enough), as $C^A(z_h, H_A) < C^{NA}(z_h, H_A)$ for all $H_A$ and $z_h \neq 0$, it holds that

$$C^A(z_h, H) < C^A(z_h, H - 1) < \ldots < C^A(z_h, 1) < C^A(z_h, 0) < \ldots \quad (14)$$

$$< C^{NA}(z_h, 0) < C^{NA}(z_h, 1) < \ldots < C^{NA}(z_h, H)$$

for all $z_h \neq 0$.

Hence, (13) holds and therefore, there is an equilibrium in which all the noise traders whose size order is not null preannounce.

It is important to point out that (14) is a sufficient condition for the existence of this sort of equilibrium. A weaker condition that also guarantees its existence is given by

$$\max_{H_A \in \{0, \ldots, H\}} C^A(z_h, H_A + 1) < C^{NA}(z_h, 0)$$

for all $z_h \neq 0$, $h = 1, \ldots, H$.

In general, other types of equilibria can exist. For instance, if there exists a value of $H_A$ such that (13) does not hold, then there is an equilibrium in which $\tilde{H}_A$ noise traders decide to preannounce and the remainder does not, where $\tilde{H}_A$ denotes the lowest $H_A$ such that (13) does not hold.

---

Footnotes:

23 The intuition of this fact is as follows. Notice that the degree of market transparency increases with the number of announcers. Then, in a market with more announcers, an increase in the price of the risky asset makes agents more optimistic about its liquidation value, which leads to a smaller reduction in the individual demands, as compared to a market with a lower number of announcers.

24 The reason of this inconclusiveness is that an increase in $H_A$ decreases both the numerator and the denominator of $C^A(z_h, H_A)$. By constrast, as the numerator of $C^{NA}(z_h, H_A)$ is independent of $H_A$ and its denominator is decreasing in $H_A$, we unambiguously conclude that an increase in $H_A$ increases $C^{NA}(z_h, H_A)$.
Next, we include two examples illustrating the aforementioned types of equilibria.

**Example 1:** $H = 2$, $z_1 = 10$, $z_2 = 10$, $N = 10$, $\phi = 5$, $\sigma_v^2 = 10$, $\sigma_e^2 = 10$, $\rho = 1$, $\sigma_2^2 = 10$.

Figure 5 shows that the graph of the function $C^{NA}(z_h, H_A)$ (blue curve) is located above the graph of the function $C^{A}(z_h, H_A + 1)$ (red curve), $h = 1, 2$. This implies that there is an equilibrium in which all the noise traders preannounce.

**Example 2:** $H = 2$, $z_1 = 10$, $z_2 = 10$, $N = 4$, $\phi = 3$, $\sigma_v^2 = 10$, $\sigma_e^2 = 10$, $\rho = 1$, $\sigma_2^2 = 10$.

Figure 6 shows that the graph of the function $C^{NA}(z_h, H_A)$ (blue curve) and the graph of the function $C^{A}(z_h, H_A + 1)$ (red curve) intersect in a point. This implies that there is an equilibrium in which one noise trader preannounces and the other noise trader does not.

$^{25}$Note that in this example the size of the orders of the noise traders coincide ($z_1 = z_2$). Then, Figure 5 can be used for both of them.
Finally, the following result shows that in large markets the decision of pre-announcement is unambiguous:

**Proposition 5:** In large markets (N or H high enough), all the noise traders (whose order size is not null) decide to preannounce their order size.

The logical for this result is as follows: In large markets the action of a noise trader has a negligible impact on the economy. In particular, in this case \(\alpha(H_A + 1)\) is very close to \(\alpha(H_A)\), making \(\gamma(H_A + 1) - \gamma(H_A)\) very small, so that the first effect of preannouncement is little relevant.\(^26\) Therefore, the second effect dominates and, consequently, all the noise traders (whose order size is not null) wish to become announcers.

6 Conclusions

We have examined the effects of disclosing information about the price-insensitive component of the order flow on the market quality. Initially, we have assumed that the decision of opaqueness/transparency is taken by the exchange and, consequently, we have compared a fully opaque market with a fully transparent market. In large markets we have obtained that transparency increases liquidity and reduces price volatility, whereas in thin markets the implications of market transparency depend on parameter specification. We have shown that the inverse relationship between price volatility and market liquidity obtained in Madhavan (1996), assuming improper priors, may not hold with proper priors. The practical implication is that a change in transparency that lowers price volatility does not always reduce the execution costs of liquidity traders.

\(^{26}\)See Comment 4 of this Section in page 18.
Our work might be relevant for researchers and regulators alike. From our
analysis it can be argued that transparency is beneficial for active securities,
independently of the public knowledge traders initially hold on their liquidation
value. For inactive securities, one market structure might turn out to be “supe-
rior” in markets with a low number of analysts or in those in which traders have
little knowledge on the securities they are trading (for instance international
traders who often lack of any expertise in the financial markets in which they
operate). Otherwise, a clear ranking between transparent and opaque markets
might not exist.

Finally, we have assumed that the decision to reveal the orders (prior to
trading) is done voluntarily by the noise traders. We obtain that the choice
of sunshine trading/dark rooms for a noise trader is independent of his order
size, being the traders with higher liquidity needs more interested in sunshine
trading, as long as this practice is desirable. Moreover, our analysis indicates
that in large markets all the noise traders opt for sunshine trading.

7 Appendix

Derivation of Equations (2) and (3). Conditional on his information set, agent
$n$ chooses $m_n$ shares of the riskless asset and $Q_n$ shares of the risky asset so as to

$$\max_{Q_n, m_n} E(e^{-\rho(Q_n + m_n)} | p, I_n)$$

$$s.t. pQ_n + m_n = p\omega_n + y_n,$$

or, equivalently, as

$$m_n = p(\omega_n - Q_n) + y_n,$$

$$\max_{Q_n} E(e^{-\rho W_n} | p, I_n)$$

$$s.t. W_n = vQ_n + p(\omega_n - Q_n) + y_n.$$ 

Let $q_n$ denote the units of the risky asset traded by investor $n$. Thus, $q_n = Q_n - \omega_n$. The previous optimization problem is equivalent to

$$\max_{q_n} E(e^{-\rho W_n} | p, I_n)$$

$$s.t. W_n = (v - p) q_n + v\omega_n + y_n.$$ 

(15)

Suppose that investors, other than $n$, use identical linear net demands given by

$$q_j(p; I_j) = \mu + \beta s_j - \alpha \omega_j - \gamma p,$$

for all $j \neq n$.

Then, from the market clearing condition, it follows that investor $n$ faces a linear
residual supply curve given by

$$p = p_n + \lambda q_n,$$

(16)
in which
\[ p_n = \frac{(N - 1) \mu + \beta \sum_{j \neq n} s_j - \alpha \sum_{j \neq n} \omega_j + z}{(N - 1) \gamma} \quad \text{and} \quad \lambda = \frac{1}{(N - 1) \gamma}. \]

Hence, investor \( n \) has to choose the quantity she trades so as to maximize her expected utility conditional on \((p_n, I_n)\). Furthermore, as \( W_n|p_n, I_n \) is normally distributed it follows that
\[ E(-e^{-\rho W_n}|p_n, I_n) = -e^{-\rho(E(W_n|p_n, I_n) - \frac{1}{2} \text{var}(W_n|p_n, I_n))}. \]

Her optimization problem simplifies to
\[ \max_{q_n} E(W_n|p_n, I_n) - \frac{\rho}{2} \text{var}(W_n|p_n, I_n), \]
with, recall (15),
\[ E(W_n|p_n, I_n) = (E(v|p_n, I_n) - p_n - \lambda q_n) q_n + E(v|p_n, I_n) \omega_n + y_n, \]
\[ \text{var}(W_n|p_n, I_n) = \var(v|p_n, I_n) (q_n + \omega_n)^2. \]

Substituting the two expressions above into the previous optimization problem and maximizing with respect to \( q_n \), the first and the second order condition require respectively, that the equality and inequality stated below hold:
\[ E(v|p_n, I_n) - p_n - 2\lambda q_n - \rho \text{var}(v|p_n, I_n) (q_n + \omega_n) = 0, \]
\[ -2\lambda - \rho \text{var}(p_n, I_n) < 0. \]

From (16) and the first equality, the value of \( q_n \) is deduced, with
\[ q_n = \frac{E(v|p_n, I_n) - p_n - \lambda q_n - \rho \text{var}(v|p_n, I_n) \omega_n}{\lambda + \rho \text{var}(p_n, I_n)}. \]

Finally, taking into account the expression of the residual supply curve, given in (16), the two (in)-equalities are equivalent to Equations (2) and (3) in the main text of the paper, as
\[ q_n(p; I_n) = \frac{E(\tilde{v}|p, I_n) - p - \rho \text{var}(\tilde{v}|p, I_n) \omega_n}{(N - 1) \gamma} + \frac{1}{\rho \text{var}(\tilde{v}|p, I_n)} \quad \text{and} \quad 2 \frac{1}{(N - 1) \gamma} + \rho \text{var}(\tilde{v}|p, I_n) > 0. \]

Proof of Lemma 1: From the market clearing condition, it follows that the vector \((p, I_n)\) is informatively equivalent to the vector \((h_n, s_n)\),
\[ h_n = \beta \sum_{j \neq n} s_j - \alpha \sum_{j \neq n} \omega_j + z, \]
so that $E(\bar{v}|p, I_n) = E(\bar{v}|h_n, s_n)$, and $\text{var}(\bar{v}|p, I_n) = \text{var}(\bar{v}|h_n, s_n)$. Hence,

\[
E(\bar{v}|p, I_n) = \bar{v} + a_1(N (\gamma p - \mu) - \beta s_n + \alpha \omega_n - \beta(N-1)\bar{v} - \bar{\tau}) + a_2(s_n - \bar{v}) \quad \text{and} \quad \text{var}(\bar{v}|p, s_n) = a_2\sigma_v^2,
\]

where

\[
a_1 = \frac{\sigma_v^2 \sigma_\omega^2 (N-1)}{D}, \quad a_2 = \frac{\sigma_v^2 (\beta^2(N-1)\sigma_\omega^2 + \alpha^2(N-1)\sigma_\omega^2 + \sigma_x^2)}{D}, \quad \text{and} \quad D = \beta^2(N-1)\sigma_x^2 + (\alpha^2(N-1)\sigma_\omega^2 + \sigma_x^2)(\sigma_v^2 + \sigma_x^2).
\]

Plugging the expression for $E(\bar{v}|p, I_n)$ obtained above into (2), and equating coefficients according to (1), it follows that:

\[
\mu = \frac{\bar{v}(1-a_2) - a_1(\beta(N-1)\bar{v} + \bar{\tau} + N\mu)}{\frac{1}{(N-1)\gamma} + \rho \text{var}(\bar{v}|p, I_n)}, \quad (17)
\]

\[
\beta = \frac{a_2 - a_1 \beta}{\frac{1}{(N-1)\gamma} + \rho \text{var}(\bar{v}|p, I_n)}, \quad (18)
\]

\[
\alpha = \frac{-a_1 \alpha + \rho \text{var}(\bar{v}|p, I_n)}{\frac{1}{(N-1)\gamma} + \rho \text{var}(\bar{v}|p, I_n)} \quad \text{and} \quad (19)
\]

\[
\gamma = \frac{1 - a_1 N \gamma}{\frac{1}{(N-1)\gamma} + \rho \text{var}(\bar{v}|p, I_n)}. \quad (20)
\]

From system above we obtain all coefficients as functions of $\alpha$. To do so, we first derive two auxiliary equations. The first one is obtained after computing $(1 - \alpha) / \gamma$, using (19) and (20), and performing simple manipulations, which gives

\[
a_1 \gamma (N - (N - 1) \alpha) = \frac{N - 2}{N - 1} - \alpha. \quad (21)
\]

The second one follows from rearranging (20), and combining it with (21), resulting in

\[
\rho \text{var}(\bar{v}|p, I_n) = \frac{2\alpha}{\gamma (N - (N - 1) \alpha)}. \quad (22)
\]

For getting $\beta$ we first divide (18) and (19) and, then, using the fact that $\text{var}(\bar{v}|p, I_n) = a_2\sigma_x^2$ in the resulting equation, we obtain the expression given in (5).

To derive the expression of $\gamma$, we first compute $\text{var}(\bar{v}|p, I_n)$. Substituting the values of $a_1$ and $a_2$ into (18), and using (22) in the resulting equation, we get

\[
\beta = \frac{\sigma_v^2 \gamma (\alpha^2(N-1)\sigma_\omega^2 + \sigma_x^2)}{\left(\frac{1}{N-1} + \frac{2\alpha}{N - (N-1)\alpha}\right) D}. \quad (23)
\]
Multiplying both sides of the previous equality by $\beta \sigma_z^2$ and taking into account the expression of $a_1$
\[
\beta^2 \sigma_z^2 = \frac{\gamma a_1 \left( \alpha^2 (N-1) \sigma_\omega^2 + \sigma_z^2 \right)}{1 + \frac{2\alpha(N-1)}{N(N-1)}} ,
\]
and from (21), it follows that
\[
\beta^2 \sigma_z^2 = \frac{\left( \frac{N-2}{N-1} - \alpha \right) \left( \alpha^2 (N-1) \sigma_\omega^2 + \sigma_z^2 \right)}{N + \alpha(N-1)} .
\]
Using the previous equality in the expression of $\text{var}(\bar{v}|p, I_n)$, we get $\text{var}(\bar{v}|p, I_n) = \frac{2\alpha^2 \sigma_z^2}{(N-1)\sigma_\omega^2 + 2\sigma_z^2}$. Substituting this expression in (22) and operating, the expression for $\gamma$ given in (6) is derived. Finally, $\mu$ as given in (4), is obtained by dividing (17) by (20), which gives
\[
\mu = \frac{\bar{v}(1-a_2) - a_1 \beta(N-1)\bar{v} + \bar{z} + N\mu}{1 - a_1N\gamma} ,
\]
which combined with (22) and $\text{var}(\bar{v}|p, I_n) = \sigma_z^2(1-a_1\beta(N-1)-a_2)$, it follows that
\[
\mu = \frac{2\alpha}{\rho \sigma_z^2 (N - (N - 1) \alpha)} - \gamma a_1 \bar{z} ,
\]
and from (21), we obtain (4).

**Proof of Lemma 2**: From (23) and (22) we obtain
\[
\beta = \frac{\sigma_z^2 \left( \alpha^2 (N-1) \sigma_\omega^2 + \sigma_z^2 \right)}{\left( \frac{1}{N-1} + \rho \text{var}(\bar{v}|p, I_n) \right) D}.
\]
Since (3) implies $\beta > 0$, it follows that $\alpha > 0$ because of (5). The inequality $\alpha < \frac{N-2}{N-1}$ follows from (24).\footnote{Note that $0 < \alpha < \frac{N-2}{N-1}$ implies that $N > 2$ must hold for a SLE to exist.} Combining this results with (6) we have that $\gamma > 0$. To derive $\alpha$ we first note that (21) and (22) imply $(N - 2 - \alpha(N - 1)) \rho \text{var}(\bar{v}|p, I_n) = 2(N - 1) a_1 \alpha$. Since $\text{var}(\bar{v}|p, I_n) = a_2 \sigma_z^2$, plugging the values of $a_1$ and $a_2$ into the previous expressions, substituting the expression of $\beta$ given in (5) in the resulting equation, straightforward computations imply that $\alpha$ is a solution of a polynomial of degree three

\[
Q(\alpha) = a_0a^3 + ba^2 + ca - d ,
\]
where $a$, $b$, $c$ and $d$ are given in the statement of this lemma.

**Proof of Proposition 1**: Lemmas 1 and 2 imply that to study the existence of a SLE we only need to analyze the roots of $Q(\alpha)$ belonging to $\left( 0, \frac{N-2}{N-1} \right)$. Next, we distinguish two cases: 1) $\sigma_z^2 \neq 0$ and 2) $\sigma_z^2 = 0$.\footnote{Note that $0 < \alpha < \frac{N-2}{N-1}$ implies that $N > 2$ must hold for a SLE to exist.}
**Case 1:** $\sigma_z^2 \neq 0$ (or equivalently $\varphi \neq 0$). Notice that $Q(\alpha) = 0$ can be rewritten as

$$(N - 1)(1 + \phi)(\alpha^2 - \phi(N - 2)N\alpha + \varphi(N - 2)N\alpha - \alpha) = 0.$$ 

As $\varphi > 0$ and $0 < \alpha < \frac{N - 2}{N - 1}$, we have that $\alpha > \frac{\phi(N - 2)N}{(\phi + 1)(N - 1)}$. Therefore, in this case $\alpha \in I = (\alpha, \frac{N - 2}{N - 1})$, where $\alpha = \max \left\{ 0, \frac{\phi(N - 2)N}{(\phi + 1)(N - 1)} \right\}$. Evaluating this polynomial in the extremes of the interval $I$, we have $Q(\alpha) < 0$ and $Q \left( \frac{N - 2}{N - 1} \right) > 0$, which guarantees existence of a zero in $I$. Finally, uniqueness follows from the fact that $Q(\alpha)$ is strictly increasing in $I$.

**Case 2:** $\sigma_z^2 = 0$ (or equivalently $\varphi = 0$). In this case, the coefficients $c$ and $d$ of the polynomial $Q(\alpha)$ are null and, therefore, we can explicitly solve the roots of $Q(\alpha)$. In this case, we get $\alpha = 0$ and $\alpha = \frac{\phi(N - 2)N}{(\phi + 1)(N - 1)}$. Then, since we need the roots belonging to the interval $I$, we conclude that in this case the existence of a SLE is guaranteed if and only if $N < (N - 2)\phi$ and in this case $\alpha = \frac{\phi(N - 2)N}{(\phi + 1)(N - 1)}$.

**Proof of Corollary 3:** In the opaque market $\varphi > 0$. From the proof of Proposition 1, we obtain that $\alpha^O > \frac{\phi(N - 2)N}{(\phi + 1)(N - 1)} = \alpha^T$. Finally, since $\beta$ and $\gamma$ as functions of $\alpha$ are increasing, we conclude that $\beta^T < \beta^O$ and $\gamma^T < \gamma^O$.

**Proof of Proposition 2:** In equilibrium, we have $\text{var}(\tilde{v}|p, I_n) = \frac{2\sigma_v^2\sigma^2}{\sigma^2(N - (N + 1)\alpha) + 2\sigma^2}$. Thus, $\text{var}^{-1}(\tilde{v}|p, I_n) = \frac{\sigma^2(N - (N + 1)\alpha) + 2\sigma^2}{2\sigma_v^2\sigma^2}$. As $\alpha^T < \alpha^O$, it follows that $\text{var}^{-1}(\tilde{v}|p, I_n)$ is higher in the transparent market.

**Proof of Proposition 3:** As all random variables are normally distributed we have that

$$\text{var}^{-1}(\tilde{v}|p^M) = \left( \sigma_v^2 - \frac{\text{cov}(\tilde{v}, \tilde{p}^M)}{\text{var}(\tilde{p}^M)} \right)^{-1}, M = O, T.$$ 

From Equations (7) and (8), we have

$$\text{var}^{-1}(\tilde{v}|p^O) = \frac{1}{\sigma_v^2} \left( \sigma_v^2 + \frac{N\sigma_v^2}{\phi + 1 + \frac{1}{N(\alpha^O)^2\varphi}} \right),$$

and

$$\text{var}^{-1}(\tilde{v}|p^T) = \frac{1}{\sigma_v^2} \left( \sigma_v^2 + \frac{N\sigma_v^2}{\phi + 1 + \frac{1 - N\mu_1^T}{N(\alpha^T)^2\varphi}} \right).$$
Notice that \( \text{var}^{-1}(\hat{\sigma}|\hat{P}) < \text{var}^{-1}(\hat{\varphi}|\hat{P}) \) is equivalent to \( 1/\alpha^O > (1 - N\mu_1^T)/\alpha^T \). Substituting \( \alpha^T \) and \( \mu_1^T \) by their values, this inequality simplifies to

\[
\alpha^O < \frac{\alpha^T}{1 - N\mu_1^T} = \frac{(N + \phi)(\phi (N - 2) - N)}{\phi(\phi + 1)(N - 1)}.
\]

It is easy to see that \( \frac{\alpha^T}{1 - N\mu_1^T} > \frac{N - 2}{N - 1} \) iff \( \phi((N - 2)^2 - 2) - N^2 > 0 \). As \( \alpha^O < \frac{N - 2}{N - 1} \), then \( \alpha^O < \frac{\alpha^T}{1 - N\mu_1^T} \) holds in this case, and consequently, prices are more informative in the transparent market. Finally, this result is also satisfied provided that \( \alpha^O \) is low enough, which is equivalent to saying that \( \sigma_2^2 \) is low enough. Just note that \( \frac{\alpha^T}{1 - N\mu_1^T} > \alpha^T \) and \( \sigma_2^2 \) low enough guarantees that \( \alpha^O \) gets close to \( \alpha^T \).

Proof of Proposition 4: From (7) and (8), it follows that

\[
\left( \frac{\partial \hat{\sigma}^O}{\partial \hat{z}} \right)^{-1} = N\gamma^O \quad \text{and} \quad \left( \frac{\partial \hat{\sigma}^T}{\partial \hat{z}} \right)^{-1} = \frac{\frac{N\gamma^T}{\hat{\sigma}_1^T + 1}}{\phi}.
\]

Substituting the equilibrium values we have that the difference in liquidity \( DL \) among the two markets is given by

\[
DL = \frac{N}{\rho_2^2} \left( \alpha^O \left( 1 + \frac{2r}{N - (N - 1)\alpha^O} \right) - \frac{(N + r(1 + \phi) + \phi)((N - 2)\phi - N)}{(\phi + 1)(N - 1)\phi} \right).
\]

where \( r = (\sigma_2^2/\sigma_1^2) \). \( DL \) is strictly increasing in \( \alpha^O \). At \( \alpha^O = \alpha^T \), it attains a negative value. At \( \alpha^O = (N - 2)/(N - 1) \) it is also negative if \( \frac{\alpha^T(1 + \phi)}{N^2\gamma^T} + \left( 1 - \phi(2 - 4N + N^2) \right) < 0 \), so that \( DL < 0 \), and the transparent market is deeper. Otherwise, the result follows when \( \sigma_2^2 \) is low enough as then \( \alpha^O \) is close enough to \( \alpha^T \).

Lemma A.1: Suppose that \( z_h \neq 0 \). Then, \( C^{NA}(z_h, H_A) \) is increasing in \( H_A \), whereas the shape of \( C^{A}(z_h, H_A) \) depends on the parameter configuration.

Proof of Lemma A.1: Applying the chain’s rule we have

\[
\frac{\partial C^{NA}}{\partial H_A}(z_h, H_A) = -\rho_2^2 \left( N - (N - 1)\alpha \right)^2 + 2N\frac{\sigma_2^2}{\sigma_1^2} \frac{\partial^2 \alpha}{\partial \varphi} \frac{\partial \varphi}{\partial H_A} \quad \text{and}
\]

\[
\frac{\partial C^{A}}{\partial H_A}(z_h, H_A) = \rho_2^2 \left( N + \alpha (N - 1) \right)^2 - 2N \left( \frac{\sigma_2^2}{\sigma_1^2} + N \right) \frac{\partial^2 \alpha}{\partial \varphi} \frac{\partial \varphi}{\partial H_A}.
\]

As \( \frac{\partial \alpha}{\partial \varphi} > 0 \) and \( \frac{\partial \varphi}{\partial H_A} < 0 \), it follows that \( C^{NA}(z_h, H_A) \) is increasing in \( H_A \), whereas

\[
\text{sign} \left( \frac{\partial C^{A}}{\partial H_A}(z_h, H_A) \right) = -\text{sign} \left( (N + \alpha (N - 1))^2 - 2N \left( \frac{\sigma_2^2}{\sigma_1^2} + N \right) \right).
\]
Next, we distinguish three cases:

**Case 1:** $4 \phi^2 \frac{(N-1)^2}{(\phi+1)^2} - 2N \left( \frac{\sigma_z^2}{\sigma_c^2} + N \right) \geq 0$ (i.e., $\phi$ high enough). In this case $(N + \alpha (N - 1))^2 - 2N \left( \frac{\sigma_z^2}{\sigma_c^2} + N \right) > 0$ for all $\alpha \in \left( \frac{N-2}{N-1}, 1 \right)$, which implies that $C^A(z_h, H_A)$ is decreasing in $H_A$.

**Case 2:** $4 (N - 1)^2 - 2N \left( \frac{\sigma_z^2}{\sigma_c^2} + N \right) \leq 0$. In this case $(N + \alpha (N - 1))^2 - 2N \left( \frac{\sigma_z^2}{\sigma_c^2} + N \right) < 0$ for all $\alpha \in \left( \frac{N-2}{N-1}, 1 \right)$, which implies that $C^A(z_h, H_A)$ is increasing in $H_A$.

**Case 3:** $4 \phi^2 \frac{(N-1)^2}{(\phi+1)^2} - 2N \left( \frac{\sigma_z^2}{\sigma_c^2} + N \right) < 0$ and $4 (N - 1)^2 - 2N \left( \frac{\sigma_z^2}{\sigma_c^2} + N \right) > 0$. In this case $C^A(z_h, H_A)$ is not monotonic in $H_A$.

**Proof of Proposition 5:** Suppose that $z_h \neq 0$. Note that $C^A(z_h, H_A + 1) < C^{NA}(z_h, H_A)$ is equivalent to

$$1 - N \mu_1 (H_A + 1) < \frac{\gamma (H_A + 1)}{\gamma (H_A)}.$$  

Direct computations yield $\lim_{N \to \infty} \alpha (H_A + 1) = \lim_{N \to \infty} \alpha (H_A) = \frac{\phi-1}{\phi+1}$. Substituting the expressions of $\mu_1 (H_A + 1)$, $\gamma (H_A + 1)$ and $\gamma (H_A)$ in (25), and taking the limit when $N$ converges to infinity, we have that $0 < 1$. Therefore, when $N$ is high enough, $C^A(z_h, H_A + 1) < C^{NA}(z_h, H_A)$ for all $H_A = 0, ..., H - 1$, and for all $h = 1, ..., H$ such that $z_h \neq 0$. This implies that all the noise traders whose order size is not null decide to preannounce their order size.

A similar reasoning can be done when $H$ is large enough. Concretely, $\lim_{H \to \infty} \alpha (H_A + 1) = \lim_{H \to \infty} \alpha (H_A) = \alpha^O$. Again, substituting the expressions of $\mu_1 (H_A + 1)$, $\gamma (H_A + 1)$ and $\gamma (H_A)$ in (25), and taking the limit when $H$ converges to infinity, we have that $1 - N \frac{N-2}{N-(N-1)\alpha^O} < 1$ as $\alpha^O < \frac{N-2}{N-1}$, the previous inequality holds. Therefore, when $H$ is high enough, $C^A(z_h, H_A + 1) < C^{NA}(z_h, H_A)$ for all $H_A = 0, ..., H - 1$, and for all $h = 1, ..., H$ such that $z_h \neq 0$. This implies that all the noise traders whose order size is not null decide to preannounce their order size.

**References**


